

MATHEMATICAL METHODS IN THE PHYSICAL SCIENCES

Third Edition

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TO THE STUDENT

As you start each topic in this book, you will no doubt wonder and ask “Just why should I study this subject and what use does it have in applications?” There is a story about a young mathematics instructor who asked an older professor “What do you say when students ask about the practical applications of some mathematical topic?” The experienced professor said “I tell them!” This text tries to follow that advice. However, you must on your part be reasonable in your request. It is not possible in one book or course to cover both the mathematical methods and very many detailed applications of them. You will have to be content with some information as to the areas of application of each topic and some of the simpler applications. In your later courses, you will then use these techniques in more advanced applications. At that point you can concentrate on the physical application instead of being distracted by learning new mathematical methods.

One point about your study of this material cannot be emphasized too strongly: To use mathematics effectively in applications, you need not just knowledge but *skill*. Skill can be obtained only through practice. You can obtain a certain superficial *knowledge* of mathematics by listening to lectures, but you cannot obtain *skill* this way. How many students have I heard say “It looks so easy when you do it,” or “I understand it but I can’t do the problems!” Such statements show lack of practice and consequent lack of skill. The *only* way to develop the skill necessary to use this material in your later courses is to practice by solving many problems. Always study with pencil and paper at hand. Don’t just read through a solved problem—try to do it yourself! Then solve some similar ones from the problem set for that section,

trying to choose the most appropriate method from the solved examples. See the Answers to Selected Problems and check your answers to any problems listed there. If you meet an unfamiliar term, look for it in the Index (or in a dictionary if it is nontechnical).

My students tell me that one of my most frequent comments to them is “You’re working too hard.” There is no merit in spending hours producing a solution to a problem that can be done by a better method in a few minutes. Please ignore anyone who disparages problem-solving techniques as “tricks” or “shortcuts.” You will find that the more able you are to choose effective methods of solving problems in your science courses, the easier it will be for you to master new material. But this means practice, practice, practice! The *only* way to learn to solve problems is to solve problems. In this text, you will find both drill problems and harder, more challenging problems. You should not feel satisfied with your study of a chapter until you can solve a reasonable number of these problems.

You may be thinking “I don’t really need to study this—my computer will solve all these problems for me.” Now Computer Algebra Systems are wonderful—as you know, they save you a lot of laborious calculation and quickly plot graphs which clarify a problem. But a computer is a tool; *you* are the one in charge. A very perceptive student recently said to me (about the use of a computer for a special project): “*First* you learn how to do it; *then* you see what the computer can do to make it easier.” Quite so! A very effective way to study a new technique is to do some simple problems by hand in order to understand the process, and compare your results with a computer solution. You will then be better able to use the method to set up and solve similar more complicated applied problems in your advanced courses. So, in one problem set after another, I will remind you that the point of solving some simple problems is not to get an answer (which a computer will easily supply) but rather to learn the ideas and techniques which will be so useful in your later courses.

M. L. B.

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