9.W Electromagnetic Waves in Vacuum

Instructions:
- □ Work in groups of 2 or 3 (or by yourself if you prefer).
- □ Work out loud on whiteboards (even if working by yourself). Take pictures of your whiteboards so you can write up the solution to include in your EM PSN as part of EM PS#14.
- □ Feel free to consult your textbook and reading/lecture notes, BUT make sure you can answer each question below with a reasonable minimum of referring to prepared notes (certainly by the end of the associated problem set and to prepare for the associated qu¡xam).
- □ Please consult with neighboring groups and with me frequently and eagerly.
- □ When completing your EM PSN Log for EM PS#14, refer to this as 9.W. There won't be a CSS solution to compare it with, but the answers are either in the book or self-correcting.

1. Write down Maxwell's equations.

2. Write down Maxwell's equations in vacuum.

3. The three-dimensional wave equation for the electric field part of an electromagnetic wave is

$$\nabla^2 \vec{\boldsymbol{E}} = \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\vec{\boldsymbol{E}}.$$

- Starting from Maxwell's equations in vacuum, derive this wave equation using vector calculus.
- In particular, start from a Maxwell's equation involving curl, and take the curl of both sides. Then, use vector identity (11) from Griffiths' front cover.
- Show all the steps of the derivation.
- Determine/write down the relationship between v and constants from Maxwell's equations.

For the remainder of this worksheet, consider a plane-polarized electromagnetic wave that has electric field part given by

$$\vec{\boldsymbol{E}} = E_0 e^{i(kx+\omega t)}\hat{\mathbf{z}}.$$

4. Show by explicit calculation that this wave function is a solution to the wave equation. Determine/write down the relationship between v and any constants in the wave function.

5. What is the polarization direction of this wave? What direction is it moving? How do you know?

6. Starting from a Maxwell's equation, figure out the magnetic field part of this EM wave.

7. Check your answer from 6. by seeing whether $\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ are transverse and give the correct direction of propagation. Also check if reproduced the result of eq (9.47).

8. Calculate the contributions of the electric field and the magnetic field to the energy density of the EM wave. Compare these contributions.

9. Calculate the Poynting vector. In particular, comment on the direction of the Poynting vector.