

Name: \_\_\_\_\_

**WEEK 8 DIFFERENTIAL EQUATIONS QUIZ**  
**PSAM, Fall 2018**

- This quiz is an opportunity for you to demonstrate your personal understanding of program material. No collaboration is allowed.
- This quiz is closed-book and closed-notes, except that you may use your personally prepared 3 inch by 5 inch notecard.
- **Show your work** and explain your reasoning; you'll be evaluated on the clarity of your work and the completeness (and correctness) of your solutions.
- You have 30 minutes to work this quiz. Please work in the space provided; use the backs of the pages if you need more room. If you have a question, please ask it!

1. Consider the periodic function  $f$  with period 4 given by  $f(x) = \begin{cases} 1 & \text{if } 0 < x < 2 \\ -1 & \text{if } 2 < x < 4. \end{cases}$

(a) Some of the coefficients in the Fourier series expansion of  $f$  can be found easily, without doing any computation. Which ones are they, and why are they easy to find? Be sure to say the value(s) of those coefficients. (You may find it helpful to sketch the given function.)

(b) Now find the rest of the coefficients in the Fourier series expansion of  $f$ .

2. Consider the function  $f$  given by  $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$

(a) Write down an expression for the Fourier transform of  $f$ .

(b) Evaluate the expression you wrote down in part (a).

(c) Now use your answer from part (b) to express  $f$  as a Fourier integral (that is, as the appropriate integral of its Fourier transform). If you have time, write the expression in terms of sines and cosines instead of complex exponentials.