Physical Systems & Applied Mathematics 2018-19 Name _____
Diagnostic Exam

Evergreen A Number _____

• You may use your personal class notes and a calculus textbook on this diagnostic exam.

- Except where prohibited, you may use a calculator or equivalent for calculation purposes.
- You may not use internet or on-line resources or discuss your work with anyone. This is an individual assessment.
- **For each question (except multiple choice/fill-in-the-blank/matching), your solution must show all your work and/or explain your reasoning. You will not receive credit for work that does not appear on the diagnostic.**

By signing below, I certify that the only materials I used to complete this exam (besides paper and writing implement) were my personal course notes, a calculus textbook, and calculator or equivalent (except where specifically prohibited). I certify that I did not use any other resources (including but not limited to internet or on-line resources). I also certify that I did not discuss the questions on this diagnostic with any other person in any way (faculty Krishna Chowdary excepted).

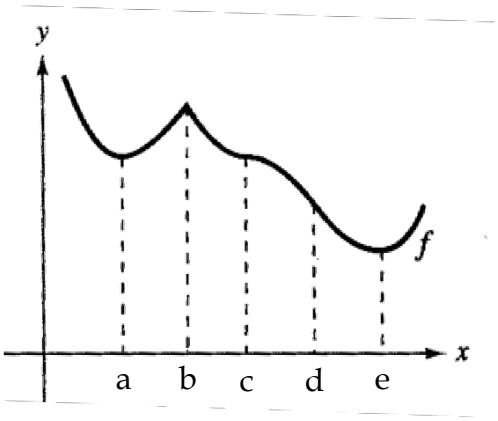
_____ _____
signature date

On completion of this Diagnostic Exam, submit it to Krishna Chowdary, using one of the following:

- (preferred) A paper copy hand-delivered to Professor Chowdary's office, Lab 2 3255, on the Olympia campus of the Evergreen State College; please slip it under the office door.
- A paper copy may be sent by physical mail to:
  Krishna Chowdary
  The Evergreen State College
  2700 Evergreen Pkwy NW
  Olympia, WA 98505
- A clear and clean scanned version of reasonable file size may be submitted as an attachment to an email to Professor Chowdary at chowdark@evergreen.edu. The scanned file should be saved as .pdf, with the file name 1819PSAMdiagnostic_YourlastnameYourfirstname.docx. So for example, if your name were Emmy Noether, you would save this with the file name 1819PSAMdiagnostic_NoetherEmmy.pdf. The subject line for your email should be PSAM Diagnostic Yourfirstname Yourlastname. Remember to actually attach the file.

- Your application is complete with both this completed diagnostic and your completed, submitted Application (instructions for completion and submission of the Application are on that document).
- All students who meet the prerequisites as demonstrated on this Diagnostic Exam and the Entrance Application will be given signature permission to register for the program. The signature will not automatically register you for the program – you will still need to register for the program. Students who receive a signature will be notified by email and expected to register within a week of the email being sent. **All qualified students will be accepted into the program**.

1. The figure below shows a graph of some function f.



The questions below are about f' and f''.

a) For which labeled values of x is $f' = 0$? Circle all that apply. If none, circle none.

a **b** **c** **d** **e** **none**

b) For which labeled values of x is $f' > 0$? Circle all that apply. If none, circle none.

a **b** **c** **d** **e** **none**

c) For which labeled values of x does f' not exist? Circle all that apply. If none, circle none.

a **b** **c** **d** **e** **none**

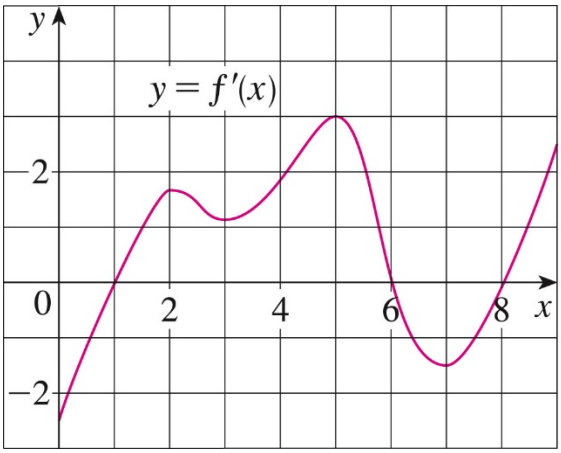
d) For which labeled values of x is $f'' = 0$? Circle all that apply. If none, circle none.

a **b** **c** **d** **e** **none**

e) For which labeled values of x is $f'' > 0$? Circle all that apply. If none, circle none.

a **b** **c** **d** **e** **none**

2. The figure to the right shows a graph of the first derivative f' of some function f (note: f is not shown)



Note: the following questions are all about f (which is not shown), not f' which is the function shown.

a) Where is f increasing? If never, write never.

b) Where is f concave up? If never, write never.

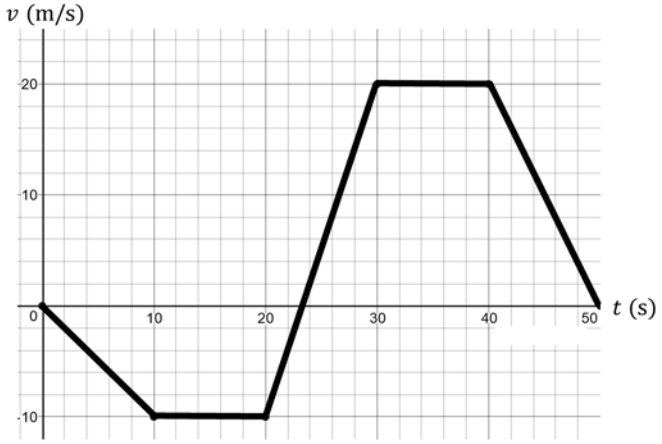
c) Where is/are the critical point(s) of f? If none, write none.

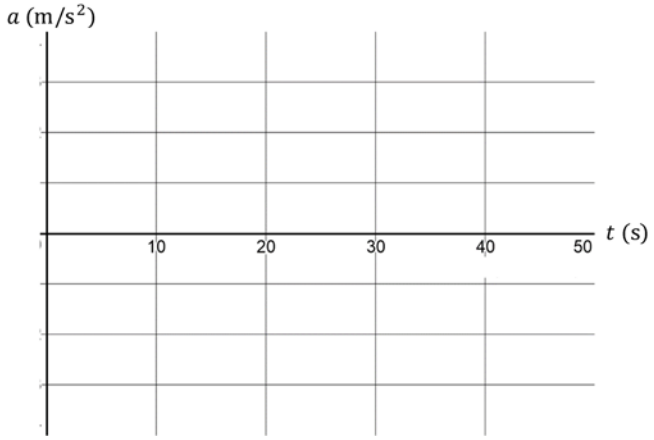
d) Where is/are the inflection point(s) of f? If none, write none.

e) Where does the slope of $f = 0$? If never, write never.

f) Where is the slope of f steepest?

3. A train moves along a straight track, with velocity v as shown in the figure. The train starts 250 m from a station. When the train moves away from the station, it has positive velocity. When the train moves towards the station, it has negative velocity.





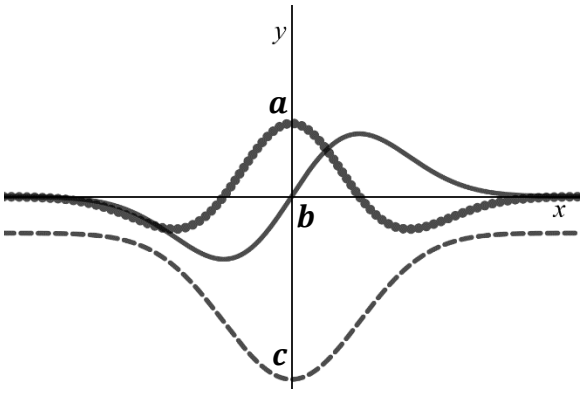
a) On the axes provided, plot the acceleration vs. time graph corresponding to the velocity vs. time graph. Make sure to include numbers on the vertical axis.

b) When is the train closest to the station? Approximately how close is it at that time? Explain your reasoning.

c) At 50 seconds, estimate how far the train is from its starting position. Explain your reasoning.

d) Between 0 seconds and 50 seconds, estimate the total distance the train has traveled. Explain your reasoning.

4) The figure to the right shows graphs of $f(x)$, $f'(x)$, and $f''(x)$. Identify each curve. Circle one choice for each.

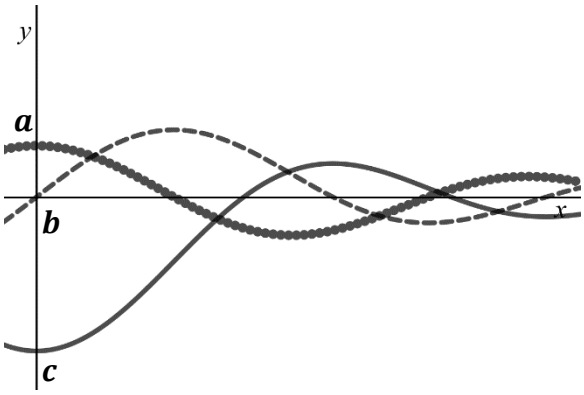


a) The graph of f is: **a** **b** **c**

b) The graph of f' is: **a** **b** **c**

c) The graph of f'' is: **a** **b** **c**

5) The figure to the right shows graphs of $f(x)$, $f'(x)$, and $\int_0^x f(t)\,dt$. Identify each curve. Circle one choice for each.



a) The graph of f is: **a** **b** **c**

b) The graph of f' is: **a** **b** **c**

c) The graph of $\int_0^x f(t)\,dt$ is: **a** **b** **c**

6) As water is drained out of a bathtub, the volume of water, V, and its height in the tub, h (measured from the bottom of the tub), change with time, t.

a) Which of the following is true of $\frac{dV}{dt}$?

positive **Zero** **negative** **not enough info to choose**

b) Which of the following is true of $\frac{dh}{dt}$?

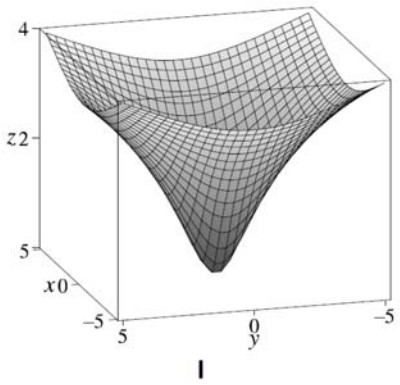
positive **zero** **negative** **not enough info to choose**

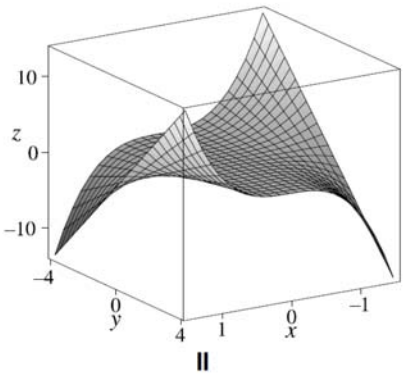
c) Which of the following is true of $\frac{dV}{dh}$?

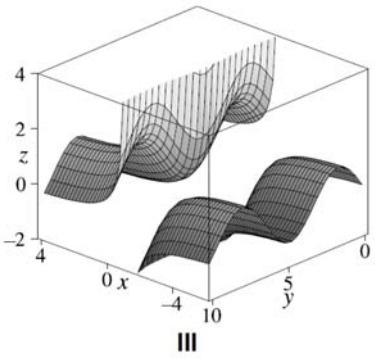
positive **zero** **negative** **not enough info to choose**

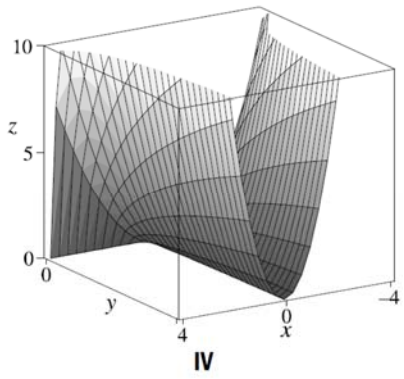
7. **Do not use a calculator, computer, or technological equivalent.** Match the equations a – f with the graphs I – VI. If you don't think the equation is represented in the graphs, circle NO MATCH.

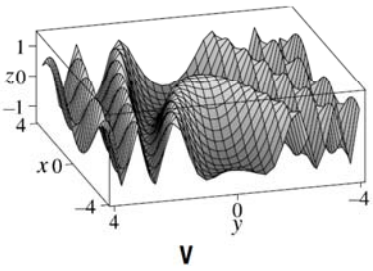
a) $f(x,y) = \dfrac{1}{x+1} + \sin y$ I II III IV V VI NO MATCH	d) $f(x,y) = \ln\left(x^2+y^2+1\right)$ I II III IV V VI NO MATCH
b) $f(x,y) = \sqrt{4-x^2-y^2}$ I II III IV V VI NO MATCH	e) $f(x,y) = x^2\sqrt{y}$ I II III IV V VI NO MATCH
c) $f(x,y) = \cos\left(x+y^2\right)$ I II III IV V VI NO MATCH	f) $f(x,y) = x^3y$ I II III IV V VI NO MATCH

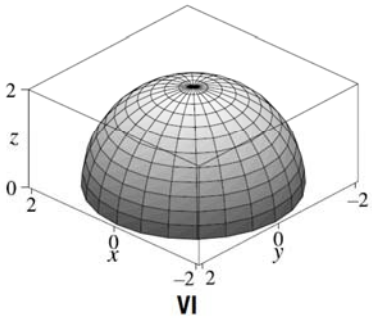












8. **Do not use a calculator, computer, or technological equivalent**; show enough work that it is clear that *you* did the calculus.

a) $f(x) = 3x^{-2} + \sin^2 x$. Find $f'(x)$.	b) $g(t) = te^{-t^2}$. Find dg/dt.
c) Find $\frac{d}{dy}(y \ln y - y)$	d) $f(x) = \int_1^x \sin(t^2)\ dt$. Find $f'(x)$.
e) $h(x) = \cos(e^{ax} + b/x)$, where a, b are constants. Find d^2h/dx^2.	f) $z(x, y) = \sin xy\ + y^2/x$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

9. **Do not use a calculator, computer, or technological equivalent**; show enough work that it is clear that *you* did the calculus. Evaluate the following.

a) $\int \frac{x+1}{x}\, dx$	b) $\int_0^1 te^{-t^2/2}\, dt$ (give exact answer)
c) $\int_0^{2\pi} \sin^2\theta\, d\theta$	d) $\int ye^y\, dy$

10. Note: you may know the unit vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ as $\hat{x}, \hat{y}, \hat{z}$.

Given $\vec{a} = 1\hat{\imath} - 2\,\hat{\jmath} + 3\hat{k}$ and $\vec{b} = -2\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$:

a) Calculate $\left|\vec{a} + \vec{b}\right|$.

b) Calculate $\vec{a} \cdot \vec{b}$.

c) Calculate $\vec{a} \times \vec{b}$.

d) Determine the angle between $\vec{a}$ and $\vec{b}$.

11. Convert between coordinate systems.

a) Convert from rectangular to cylindrical coordinates: $(x, y, z) = (1, -1, 4)$ $(r, \theta, z) = ?$

b) Convert from spherical to rectangular coordinates: $(\rho, \theta, \phi) = (2, \pi/3, \pi/4)$ $(x, y, z) = ?$

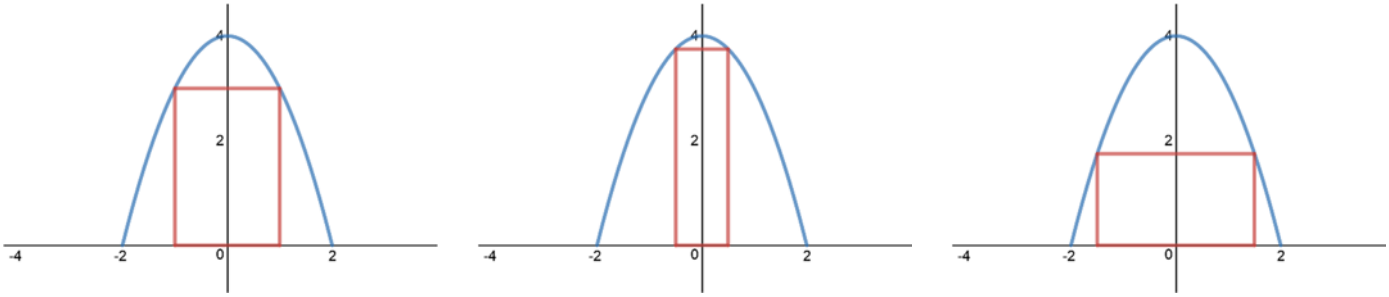
12. Given the function $f(x) = 2^x$.

a) Find $f'(x)$.

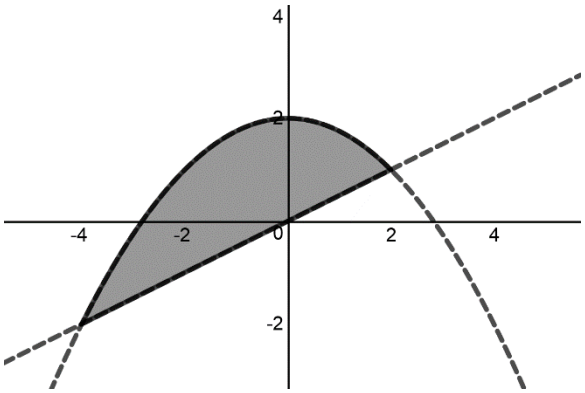
b) Find the equation of the tangent line to the function at $x = 2$.

c) Find the value of x where the slope of the tangent line to the function equals 1.

13. The part of the parabola $y = 4 - x^2$ above the x-axis is shown in the figure. A rectangle has its base on the x-axis and its height defined by having its upper corners touch the parabola. Several example rectangles are shown. Determine the maximum area of any such rectangle that can be drawn in this parabola.



14. The area between the line $y = \frac{x}{2}$ and the curve $y = 2 - \left(\frac{x}{2}\right)^2$ is shown shaded in the figure. Calculate this area.



15. Solve the differential equation

$$\frac{dx}{dt} = x - xt$$

where $x(0) = 2$.

16. Given $f(x) = (1 - x)^{1/3}$. When $x < 1$, the power series expansion of $f(x)$ converges. Use the Maclaurin series to calculate the first four terms of the power series expansion. Note: the Maclaurin series is the special case of the Taylor series centered at 0.

17. Find the equation of the plane determined by the intersecting lines L1 and L2.

L1: $x = -1 + t,\ y = 2 + t,\ z = 1 - t$ L2: $x = 1 - 4s,\ y = 1 + 2s,\ z = 2 - 2s$