

Worksheet: 9.2 ■ ELECTROMAGNETIC WAVES IN VACUUM

Instructions:

- Work in groups of 2 or 3 (or by yourself if you prefer)
- Feel free to consult your textbook and reading/lecture notes, BUT make sure you can answer each question below with a reasonable minimum of referring to textbook/notecards (certainly by the end of the associated problem set and by the associated quiz/exam).
- Work out loud on whiteboards (even if by yourself), and also get your work in your personal notes.
- Don't rush. You can finish on your own time.

1. Write down Maxwell's equations.
2. Write down Maxwell's equations in vacuum.
3. The three-dimensional wave equation for the electric field part of an electromagnetic wave is

$$\nabla^2 \vec{E} = \frac{1}{v^2} \frac{\partial^2}{\partial t^2} \vec{E}.$$

- Starting from Maxwell's equations in vacuum, derive this wave equation using vector calculus.
- Show all the steps of the derivation.
- Determine/write down the relationship between v and constants from Maxwell's equations.

For the remainder of this worksheet, consider a plane-polarized electromagnetic wave that has electric field part given by

$$\vec{E} = E_0 e^{i(kx + \omega t)} \hat{z}.$$

4. Show by explicit calculation that this is a solution to the wave equation. Determine/write down the relationship between v and any constants in the wave function.
5. What is the polarization direction of this wave? What direction is it moving? How do you know?
6. Starting from a Maxwell's equation, figure out the magnetic field part of this EM wave.
7. Check your answer from 6. by seeing whether \vec{E} and \vec{B} are transverse and give the correct direction of propagation. Also check if reproduced the result of eq (9.47).
8. Calculate the contributions of the electric field and the magnetic field to the energy density of the EM wave. Compare these contributions.
9. Find all elements of the Maxwell stress tensor for this electromagnetic wave. Since $-\vec{T}$ represents the momentum flux density, interpret the meaning of the stress tensor in this case.

Follow-up: Turn these questions/tasks into Learning Goals. For example: "I can derive the wave equation for the electric or magnetic field part of an electromagnetic wave starting from Maxwell's equations."