4.1.23

a) <u>FALSE.</u>

This would be true if

$f(t)=0$ for all t (4.1, 2nd paragraph of Ex 5)

b) <u>False.</u> We interpret arrows with a magnitude of ∞

Vector must have 3 components i.e. $\vec{v}=\begin{bmatrix}0\\1\end{bmatrix}$ not in 3d, $\vec{v}=\begin{bmatrix}0\\0\\1\end{bmatrix}$ is in 3d.

c) The 0 vector cannot move, so this is <u>True</u>

d) <u>True</u>, by definition of a subspace $\longrightarrow$ (See the subspace definition)

4.2.25

a) the null space of A is the solution set of the equation $A\vec{x}=0$

$\boxed{\text{True}}$

See definition on page 201

b) THE NULL SPACE OF AN $m \times n$ MATRIX IS IN $\mathbb{R}^m$

$\boxed{\text{FALSE}}$ NUL IS IN $\mathbb{R}^n$ # COLUMNS free variables

c) The column space of A is the range of the mapping $\vec{x} \longmapsto A\vec{x}$

$\boxed{\text{True}}$ THE RANGE OF $T(\vec{x})$ IS THE COLUMN SPACE OF A.
IF $T(\vec{x}) = A\vec{x}$ FOR SOME MATRIX A PAGE 206

d) IF THE EQUATION $A\vec{x}=\vec{b}$ IS CONSISTENT THEN Col A IS $\mathbb{R}^m$

$\boxed{\text{FALSE}}$ #8 Col $A=\mathbb{R}^m$ if <u>and only if</u> $A\vec{x}=\vec{b}$ has a solution for $\boxed{\text{EVERY } \vec{b}}$ in $\mathbb{R}^m$ P. 206

e) The kernal of a linear transformation is a vector space

$\boxed{\text{TRUE}}$
- nullspace of a matrix is a vector space p 201
- Kernel of T is the null space of A, where $T(\vec{x}) \mapsto A\vec{x}$ p206

f) Col A IS THE SET OF ALL VECTORS THAT CAN BE WRITTEN AS $A\vec{x}$ for some $\vec{x}$

$\boxed{\text{TRUE}}$ Col $A = \{\vec{b} : \vec{b} = A\vec{x} \text{ for some } \vec{x} \text{ in } \mathbb{R}^n\}$ P. 203

a) a linearly independent set in a subspace H is the basis of H

True, if H is a subspace of vector space V, then the basis of H is formed by the linearly independent set $B = \{\vec{b_1} \dots \vec{b_n}\}$ (pg 221)

ⓑ If a finite set S of Nonzero vectors spans a vector Space V, then Some subset of S is a basis for V. Spanning Set theorem (pg 212)

True, if there is a linearly independent Set

c) A basis is a linearly independent set that is large as possible (pg. 214 "Two views of a basis")

True, some subset of S that is the basis of V must contain only and all linearly independent vectors, no more, no less

ⓓ The Standard Method for Producing a spanning Set for Nul A, described in Section 4.2, Sometimes fails to produce a basis for Nul A (pg. 203, ex. 3, pg. 211)

False, the spanning Set is automatically linearly independent

e) If B is an echelon form of matrix A, then the pivot columns of B form a basis for col A (ty Krishna)

false, the pivots of B indicate which pivots of A form the basis of col A

4.3.22

15:

a) True, if $\underline{x}$ is originally in V which has some dimensionality then the basis of $\underline{x}$ will be in $\mathbb{R}^n$
Pg 218

b) False, $\underline{x} = P_{\mathcal{B}}[\underline{x}]_{\mathcal{B}}$ $P_{\mathcal{B}}\underline{x} \neq [\underline{x}]_{\mathcal{B}}$, $P_{\mathcal{B}}^{-1}\underline{x} = [\underline{x}]_{\mathcal{B}}$ Pg 221

c) False, the space $\mathbb{P}_3$ is isomorphic to $\mathbb{R}^4$.

4.4.15

4.5.19

a) True
p.230

b) True but only if the plane passes through the origin because the subspace must contain $\vec{0}$

p.229

c) False
(It depends on the basis of $\mathbb{P}_4$.) ?

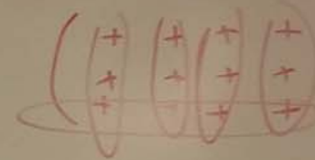
The dimension of the vector space $\mathbb{P}_4$ be 5 p.222 Example 5 and 6

d) True but only if the set of S has n elements
Theorem 12 p.229

e) True
Theorm 9 p227

4.6.17 Let A be an $m \times n$ matrix

a) The row space of A is the same as the column space of A^T



$\boxed{\text{True}}$ P. 236 2nd paragraph

b) If B is any echelon form of A, and if B has three nonzero rows, then the first three rows of A form a basis for Row A

$$A = \begin{pmatrix} 1 & 6 & 2 \\ 0 & 4 & 17 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix} \quad \text{not a Basis} \qquad B = \begin{pmatrix} 1 & 6 & 2 \\ 0 & 4 & 17 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{totally a basis}$$

$\boxed{\text{False}}$ The first three rows of B form a basis for Row A

c) dim Row A = dim Col A even if A is not square

$\boxed{\text{True}}$ by Thrm 14

d) The sum of the dimensions of the row space and the null space of A equals the number of rows of A.

$\boxed{\text{False}}$ The sum equals the number of columns of A

4.7.11 & 4.7.12

11. a) The columns of the change-of-coordinates matrix $\underset{C \leftarrow B}{P}$ are B-coordinate vectors in C

False Theorem is

"The columns $\underset{C \leftarrow B}{P}$ are the C coordinate vectors of the vectors in the basis B"

b) If $V = \mathbb{R}^n$ and C is the standard basis for V then $\underset{C \leftarrow B}{P}$ is the same as the change-of-coordinates matrix P_B introduced in section 4.4.

True

Pg 243 Change of basis in $\mathbb{R}^n$

12.

a) The columns of $\underset{C \leftarrow B}{P}$ are linearly independent

TRUE. pg 242 (under fig. 2)

b) If $V = \mathbb{R}^2$, $B = \{b_1, b_2\}$, and $C = \{c_1, c_2\}$, then row reduction of $[c_1\ c_2\ b_1\ b_2]$ to $[I\ P]$ produces a matrix P that satisfies $[X]_B = P[X]_C$ for all $\vec{x}$ in V.

FALSE. reducing $[c_1\ c_2\ b_1\ b_2]$ gives $[I\ \underset{C \leftarrow B}{P}]$

$\underset{C \leftarrow B}{P}$ would satisfy $[X]_C = P[X]_B$

(since we're transforming from B to C)