Mathematica Lab 2: Hyperbolic Functions, Solve, NDSolve, ParametricPlot

Goals: Visualize graphs of hyperbolic functions and their inverses. Get an intuitive sense of what it means to solve an equation by solving it graphically and more practice using Solve. Get more practice using NDSolve. Learn how to make parametric plots.

Part 0. Navigate to the Handouts folder in the physical-systems program file share, open the MMA folder, and copy MMA Lab 2 to your Cubbie. Open MMA Lab 2.

PART 1: Plotting hyperbolic trig functions

1. MMA Lab 2 uses comments to give some directions. Follow along with the directions in MMA Lab 2 until directed back here.

PART 2: Problem 2.22 – Linear Drag and Solve

1. Read (but don't begin!) Problem 2.22.
2. The problem statement claims that equation (2.39) cannot be solved for R analytically in terms of elementary functions. Is this claim correct? See what happens when you try to solve equation (2.39) for the range R.
3. Even though this can't be solved analytically, it can be solved graphically for particular values of parameters. Using the particular values suggested in Problem 2.22, convince yourself that equation (2.39) can be rewritten as follows:

$$\frac{\sin\theta + 1}{\cos\theta} R = -\ln\left(1 - \frac{R}{\cos\theta}\right)$$

   Note that Problem 2.22 sets $v_0 = v_{term} = g = 1$, but what happened to τ in equation (2.39)? Hint: check out equation (2.34).
4. Note that for a particular value of θ, each side of the equation above is a function of R. So we can plot each side, and look for the intersection. Go back to MMA Lab 2.
5. On your own, complete Problem 2.22 (don't complete it now, since you'll want to get the rest of the lab parts done). Summarize your results in a nice table.

PART 3: Example 2.6 – Quadratic Drag, NDSolve, and ParametricPlot

1. Our goal is to reproduce Figure 2.10 in Example 2.6 (at least the solid curve for quadratic drag, there's not much involved to get the dashed line which shows the trajectory with no air resistance).
2. Examine equations (2.61). Convince yourself that this is equivalent to the following (note that you'll need equation (2.53) as well):

$$\ddot{x} = -\frac{g}{v_{term}^2}\left(\sqrt{(\dot{x})^2 + (\dot{y})^2}\right)\dot{x}$$
$$\ddot{y} = -g - \frac{g}{v_{term}^2}\left(\sqrt{(\dot{x})^2 + (\dot{y})^2}\right)\dot{y}$$

3. This coupled nonlinear differential equation has no analytical solution. We can solve it numerically using MMA. For convenience, we'll use $g = 10$ m/s^2 and $v_{term} = 35$ m/s (as found in Example 2.5). Examine the following MMA expression, and see what sense you can make of the syntax.

```
prelimsol = NDSolve[{
    x''[t] == -(10/35^2)*Sqrt[(x'[t])^2 + (y'[t])^2]*x'[t],
    y''[t] == -10 - (10/35^2)*Sqrt[(x'[t])^2 + (y'[t])^2]*y'[t],
    x[0] == 0, x′[0] == 19.3` , y[0] == 0, y′[0] == 23},
   {x[t], y[t], x'[t], y'[t]}, {t, 0, 8}]
```

4. Return to MMA Lab 2 and complete the associated sections.

PART 4: Problem 2.55 and ParametricPlot

NOTE: This has been corrected from the version handed out during lab. I apologize for my mistake.

1. Read (but don't begin!) Problem 2.55, which is a challenging homework problem due Thu. Oct. 6.
2. The equations of motion (which you are to derive for that homework assignment) can be solved (which you are to do also as part of that homework assignment) to show that

$$x(t) = v_{dr}t + (1/\omega)(v_{x0} - v_{dr})\sin\omega t \qquad y(t) = (1/\omega)(v_{x0} - v_{dr})(\cos\omega t - 1) \qquad z(t) = \text{constant}$$

Choosing convenient constants $v_{dr} = \omega = 1$, these can be written as

$$x(t) = t + (v_{x0} - 1)\sin t \qquad y(t) = (v_{x0} - 1)(\cos t - 1) \qquad z(t) = 0$$

3. Using the help menu in MMA, review the syntax for ParametricPlot and ParametricPlot3D. Note that in this case, since $z(t) = 0$, you can just use ParametricPlot. Make nice plots of the trajectory for various values of v_{x0} (extend the time long enough that you see the trajectory repeat). You need this for the last part of Problem 2.55, so include with that assignment.

PART 5: Optional Challenge Extension. If you make progress, email Krishna so he can look at your MMA notebook.

Modify Part 3 so that you can:

- input an arbitrary velocity and angle and output the trajectory;
- show multiple trajectories on the same graph;
- automatically find the time of flight, max height, and max range.

SAVE TO YOUR CUBBIE BY FRIDAY OCTOBER 7