Mathematica Lab 1: Introduction, Simple Plots, Direction Fields, Numerical Solutions[1]

Goals: Get a very basic introduction to the computer algebra system *Mathematica* (MMA). Learn how to make basic plots and 3D plots. Learn how to plot direction fields and integral curves. Learn to use DSolve and NDSolve.

Part 0. File Share, Launching Mathematica

o Figure out how to launch Mathematica.
o (Ask your neighbor if unfamiliar with file share system) Find the program file share, called physical-systems. Navigate to the Handouts folder, open the MMA folder, and copy MMA Lab 1 to your Cubbie.
o You should be able to open MMA Lab 1 either in MMA or by double clicking on the file.

PART 1: Introduction to Mathematica

1. Some terminology: input is entered into a "cell". We can "evaluate" a cell by clicking within a cell and hitting Shift+Enter (at the same time). Output is generated in a new cell.
2. Click in the first cell, that has 2 + 2 in it. Evaluate the cell and see what changes. (You may be startled at the time it takes to do this simple calculation; whenever MMA performs its first calculation (not when it opens), it generates a "kernel". Launching the kernel is what is taking so long).
3. Evaluate that first cell again (the one with 2 + 2) in it. Note how long it takes this time, and note what changes (this might be a bit subtle, ask a neighbor if you're not sure).
4. Click in the next cell, that has Solve[3x + 1 = 4], and evaluate the cell. What happens?
5. MMA has an extensive help menu. Go to Help and either Documentation or Find Selected Function (note the keyboard shortcut), and type Solve in the search bar. Carefully compare the syntax in one of the examples to the cell that gave you trouble, and fix that cell. Evaluate it again.
6. Change that cell to Solve[2x^2 – 2x – 1 == 0], and evaluate it again. (Instead of changing the cell, you could insert a new cell by clicking right after the output for the previous cell and typing.)
7. Change that cell (or insert a new one) to Solve[3x + y == 7 && 2x – y == 1], and evaluate it again.
8. Note: you can delete input cells, output cells, or both by clicking on the brackets on the right hand of the screen, and pressing Delete.

PART 2: Simple Plots and 3D plots

1. Evaluate the cell with Plot[Sin[x], {x, 0, 6 Pi}].
2. Evaluate the cell with Plot[{Sin[x], Sin[2 x], Sin[3 x]}, {x, 0, 2 Pi}, PlotLegends -> "Expressions"].
3. Evaluate the cell with Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}]. Hover over the output graph with the pointer – what do you notice? Click **and** drag on the graph – what do you notice? Just click on the graph – now, make it bigger.
4. Evaluate the cell again. What do you notice about the output graph? Delete just the output, then evaluate the cell again. What do you notice.
5. Click and drag on the graph again. Which axis is x? y? z? Adjust the input cell to Plot3D[Sin[x + y^2], {x, -3, 3}, {y, -2, 2}, AxesLabel → Automatic], and evaluate the cell again.
6. FYI, there are many options to adjust the appearance of plots, which can be found in the Help menus. Don't take time to explore right now, but recognize that the Help feature is extremely helpful (sorry, couldn't resist).

PART 3: Direction Fields and Integral Solutions

1. Consider the differential equation $\frac{dy}{dx} = x^2$. Fill out the table for a few values of x:

x	x^2	dy/dx
-3		
-1		
0		
1		
2		
4		

2. Consider what the direction field (note: directions fields are also called slope fields) would look like. You can picture it in your imagination, or sketch it in your notes.
3. We know that $\frac{dy}{dx} = x^2$, which means that e.g., for x = 2, the slope is 4. We can understand slope to mean go over 4 in the x-direction, and go up 1 in the y-direction. We can also think of this as a vector: go over 4 in the x-direction, and go up 1 in the y-direction. Ask if this doesn't make sense.
4. MMA doesn't plot direction fields directly. We can modify the function VectorPlot to plot direction fields. Evaluate the cell with VectorPlot[{1,x^2}, {x, -2, 2}, {y, -2, 2}]. Does this seem reasonable?

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[1]Portions of this exercise are based on "Creating Slope Fields" from the Ordinary Differential Equations *Mathematica* Laboratory from San Joaquin Delta Valley College, available at http://calculuslab.deltacollege.edu/ODE/7-1/7-1-0-h.html

5. This doesn't look quite like a slope field should look like. Compare the input in the previous step to that in the next cell VectorPlot[{1, x^2}, {x, -2, 2}, {y, -2, 2}, VectorStyle -> Arrowheads[0], Axes -> True]. Hopefully the changes make sense to you. Evaluate this cell and see what you get.
6. This still doesn't look quite like the slope fields we are used to, due to the different lengths of the lines. You may also want to change the number of vectors plotted (by default, MMA plots 15 x 15). Compare the input in the previous step to that in the next cell slopefield1 = VectorPlot[{1, x^2}, {x, -2, 2}, {y, -2, 2}, VectorStyle -> Arrowheads[0], Axes -> True, VectorScale -> {Tiny, Automatic, None}, VectorPoints -> 20]. Hopefully the changes aren't too confusing. Evaluate this cell and see what you get.
7. To plot any slope field you like, you can modify this last expression. What in particular would you modify? Copy this cell and modify it to plot the slope field for $y' = 2y - 1$.
8. **IMPORTANT: Using MMA to plot slope fields is very convenient. Do not use it to substitute for your conceptual understanding and ability to sketch slope fields by hand.**
9. Next, we'll see how to plot integral curves (also called solution curves). Slope field show the slopes of the solutions y to the differential equation y'. We can use these slopes to get a sense of the shape of y. Go back to your slope field for $y' = x^2$. Can you make a guess as to what the shape of a particular y is given the slope field? Ask if you're unsure what these means.
10. We'll do this with more care in the future. For now, convince yourself that for $\frac{dy}{dx} = x^2$, it is plausible that $dy = x^2 dx$. Integrate both sides and convince yourself that $y = \frac{1}{3}x^3 + C$. Because we integrated once, we see one arbitrary constant of integration. So $y = \frac{1}{3}x^3 + C$ give an infinite family of curves, each of which depends on a different initial condition. Show that $y = \frac{1}{3}x^3 + C$ gives solutions to the differential equation $\frac{dy}{dx} = x^2$.
11. Let's consider the particular integral curve where $= 1$, so we have the particular solution $y = \frac{1}{3}x^3 + 1$. Evaluate the cell with integralcurve1 = Plot[1/3 x^3 + 1, {x, -2, 2}, PlotStyle -> {Color = Red}].
12. Evaluate the cell Show[slopefield1, integralcurve1]. Think carefully about what the slope field tells you about the solution curve.

PART 4: DSolve and NDSolve

1. We saw in the previous part that solutions to the differential equation $\frac{dy}{dx} = x^2$were given by $y = \frac{1}{3}x^3 + C$. Let's see how to get this with MMA.
2. Evaluate the cell with DSolve[{y'[x] == x^2}, y[x], x]. Hopefully this usage makes sense. What do you get?
3. Now, evaluate the cell with DSolve[{y'[x] == x^2, y[0]==1}, y[x], x]. What's different in the input? Does that make sense? Did you get the output you expected (perhaps algebraically identical)?
4. Evaluate the cell with Plot[y[x] /. %, {x, -2, 2}]. This syntax is a little tricky – don't worry about it right now.
5. Taylor Example 1.2 uses the small angle approximation to arrive to get to equation (1.55): $\ddot{\phi} = -\omega^2\phi$, where ω is defined by equation (1.54). This is a famous differential equation, and we'll get practice solving it. For now, we'll do it in MMA. Evaluate the cell with DSolve[ϕ''[t] == -ω^2 ϕ [t], ϕ [t], t]. How many arbitrary constants do you get? Does this make sense? Compare to equation (1.56), and make sure you can reconcile the notation.
6. Still with Taylor Example 1.2, the exact differential equation for this situation is $\ddot{\phi} = -\omega^2 \sin\phi$. Let's try this in MMA – evaluate the cell with DSolve[ϕ''[t] == -ω^2 Sin[ϕ [t]], ϕ [t], t]. What do you get? Taylor mentions the Jacobi elliptic functions, but those aren't very useful to us.
7. Instead, we'll try to do this with NDSolve, which will solve the differential equation numerically rather than looking for an analytic answer. In order to solve numerically, we'll need to choose initial conditions and other numerical values. Let's choose a small initial angle to see if the numeric solution looks reasonable. We'll use the following parameters: $\phi_0 = \phi(0) = 9^o = \frac{\pi}{20}, g = 9.8\frac{\mathrm{m}}{\mathrm{s}^2}, R = 5\ \mathrm{m}$. Look at the second to last cell (starts with ballRoll1) and see if the input makes sense. Evaluate this cell and the next. Does this look like what you'd expect?
8. **Modify as needed to solve Taylor 1.50 and 1.51. All the various techniques you'll need are modeled in the previous steps.**

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