Your name: __________________________

Last, First

Instructions:

1. CLOSED BOOK.

2. HONOR CODE: you must do your own work, and use no external resources (notes, etc.)

3. READ the whole test first. 100 points total.

4. STRATEGY:
   a. Do the easy problems first! Get all the points you can!
   b. Do the hard problems next; if really stuck, circle it to come back to it later, then move on to the next question. Don’t just work away all your time on one problem.

5. SHOW your work. An answer with no work = zero points.
Basics:

1. Fill in the following formulas (12 points):

| **Op-amps: Gain of an inverting amplifier.** | \[ G = -\frac{R_2}{R_1} \] |
| **Draw the circuit.** |

| **Op-amps: Gain of a non-inverting amplifier** | \[ G = \frac{R_1 + R_2}{R_2} = 1 + \frac{R_1}{R_2} \] |
| **Draw the circuit.** |

| **\( f_{3dB} \) for an RC circuit:** | \[ \omega R_C = 1 \implies f_{3dB} = \frac{1}{2\pi R_C} \] |

| **Resonant frequency for an LC circuit:** | \[ F_{res} = \frac{1}{2\pi \sqrt{LC}} \] |
2. (10 pts) Speaker cable inductance. Fancy speaker cable manufacturers want you to spend a lot of money on exotic cables – they’ll tell you the cable matters as much as the amplifier. One manufacturer sells a fancy (and expensive) cable designed to minimize inductance of the wire, on the theory that the wire inductance causes problems for audio quality.

   I checked a reputable cable manufacturer (Belden – they are a good place to go for many sorts of cable), and their good speaker wire has a inductance of $L = 0.15 \mu\text{H/foot}$. say you have 2m (6 ft) of cable to your speaker… that’d be $1 \mu\text{H}$; $2 \mu\text{H}$ including the return cable. For those interested, check the data sheet – has inductance, resistance, capacitance, etc etc etc:
   
   http://www.belden.com/techdata/english/5000UE.pdf

a. Is the “Audiophile cable” manufacturer’s claim true, or is it an expensive scam? Assume your 6’ of cable have $L = 2 \mu\text{H}$, and the speakers are purely resistive and $8\Omega$.

i. Model it as a voltage divider. What is the effect of the inductance (i.e., what are $G$, and $|G| = \frac{|V_{\text{out}}|}{V_{\text{in}}}$, for the divider?)

   \[
   V_{\text{in}} = \frac{L}{Z_{L}} \quad V_{\text{out}} = \frac{R}{Z_{L}}
   \]
   
   Voltage divider: $G = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R}{R+Z_{L}}$

   So $G = \frac{R}{R+i\omega L}$ and $|G| = \sqrt{\frac{R^2}{R^2 + (\omega L)^2}}$

   \[
   |G| = \sqrt{\frac{1}{1 + (\omega L/R)^2}}
   \]

   Effect of the inductance, $Z_L = i\omega L$, is to block high frequencies. $Z_L \to \infty$ as $\omega \to \infty$

   This is a Low-Pass filter.

ii. At what frequency, if any, will this be a problem? If there is a 3dB point, give it. Human hearing runs up to $\sim 20\text{kHz}$... do you need expensive “low inductance” cable?

   From the equation for $|G|$, we see that the 3dB point (where $|G| = \frac{1}{\sqrt{2}}$ of its peak value) will be at $\frac{\omega L}{R} = 1$, so

   \[
   F_{3\text{dB}} = \frac{R}{2\pi L}
   \]

   For $R = 8\Omega$ and $L = 2\mu\text{H}$,

   \[
   F_{3\text{dB}} = \frac{8\Omega}{2\pi \cdot 2\mu\text{H}} \approx \frac{8}{12} \approx 0.666 \text{kHz} = F_{3\text{dB}}
   \]

   This is $\gg$ than the human hearing range, so cable inductance is not really a problem here.
3. (5 pts) Power. You want to design a “100W” stereo amplifier. Most claims for 100W are scams, but you want to do the real thing... 100W into an 8Ω speaker. What maximum voltage does your amplifier need to put out? Multiply your answer by \( \sqrt{2} \), since you want to have “100W RMS output power”, i.e. power assuming a sine wave.

\[
\begin{align*}
\rho &= \frac{V^2}{R} \\
V &= \sqrt{\rho R} = \sqrt{100W \cdot 8\Omega} = \sqrt{800W} \\
&= 10 \cdot 2\sqrt{2} \approx 28V.
\end{align*}
\]

Multiply \( V \) by \( \sqrt{2} \) to get peak voltage for a sine wave (since we want 100W average power).

\[
V_{\text{peak}} = V \cdot \sqrt{2} = 10 \cdot 2\sqrt{2} \cdot \sqrt{2} = 40V = V_{\text{peak}}
\]

So you need 40V peak (\( \pm 40V \) rails) to deliver 28V rms for 100W power, \( \approx 8\Omega \).

4. (10 pts) Say you want to build an amplifier that puts out \( \pm 40V \). Audio frequencies go to 20kHz.

a. What slew rate does your amplifier need to adequately produce the music? Explain your result / show your work.

**Slew rate is the rate of change of \( V_{\text{out}} \) that an amplifier can support - \( \frac{dV_{\text{out}}}{dt} \).**

For \( V_{\text{out}} = 40V \sin wt \),

\[
\frac{dV_{\text{out}}}{dt} = 40V \cdot w \cos wt \leq 40V \cdot w
\]

since \( \cos wt \leq 1 \).

So the maximum \( \frac{dV}{dt} \) we will have is \( \frac{dV}{dt} = 40V \cdot 2\pi \cdot 20\text{kHz} = 5\text{MV/s} \).

(\( \text{since } w = 2\pi f \))

So the amplifier needs a slew rate of \( \frac{dV_{\text{out}}}{dt} = 5\text{V/\mu s} \) to adequately reproduce the music.

b. The 741 op-amp has a slew rate of \( \approx 1\text{V/\mu s} \). Is the stereo amplifier application more or less demanding than the op-amp spec?

\( 5\text{V/\mu s} \) is a lot larger than \( 1\text{V/\mu s} \) ... so in terms of required slew rate the stereo amplifier is a more demanding application.
5. (15 pts) (HHayes p186/1e) Dealing with op-amp imperfections. Op-amps are not ideal, and the small offsets they have add up and become a big problem when you make an op-amp integrating circuit (an “integrator”, as shown in (A) below.

A:

b. Plot of \(|G|\) vs the frequency \(\omega\), with any important points/features labeled. When does the resistor \(R_2\) begin to affect circuit behavior? Discuss the circuit behavior for \(R_1 = 100k\), \(R_2 = 10M\), and \(C = 1\mu F\).

c. How might this help with steady offsets in the incoming signal, \(V_{in}\)? (or equivalently due to op-amp imperfections.)
6. (12 pts) Cable termination buffer. Short pulses (say, a few nanoseconds wide) are often generated in scientific experiments – e.g., as the timing signals for pulsed laser experiments, etc. Fast signals (radio frequency, of which short pulses are a subset) generally need to be terminated into 50Ω resistances or else they reflect back along the cable and cause havoc (you start detecting multiple pulses due to the echoes, instead of just the “real” one.) You want to detect a pulse from a cable and buffer it using an op-amp, with a gain of \( |G| = 10 \) (sign doesn’t matter.)

a. Draw an inverting op-amp circuit to do this \( (|G| = 10, 50\Omega \text{ input impedance}) \):

\[
\begin{align*}
|G| & = \frac{R_2}{R_1} = 10, \quad \text{so} \quad R_2 = 500 \Omega \\
& \Rightarrow |G| = 10, \quad \text{as desired, so} \quad \begin{cases} 
R_1 = 50 \Omega \\
R_2 = 500 \Omega 
\end{cases}
\end{align*}
\]

b. Draw a non-inverting op-amp circuit to do this \( (|G| = 10, 50\Omega \text{ input impedance}) \):

\[
\frac{R_1}{R_2} + 1 = 10 \quad \Rightarrow \quad \begin{cases} 
R_1 = 9 \Omega \\
R_2 = 1 \Omega 
\end{cases}
\]
7. (10 pts) Function generator settings. For fast-pulse systems, you need 50Ω termination on cables to prevent reflections. Technically you can do this by using a 50Ω output impedance for the device you use to send the pulse out (since that will then eliminate any reflection from the far end), OR a 50Ω input impedance for the device that detects the pulse.

Of course, if you don’t know whether some customer will terminate the thing with a 50Ω, you better design your device with a 50Ω output impedance… which is why most function generators have 50Ω output impedance. But if you (the experimenter) don’t know if the person who designed the signaling device gave it a 50Ω output impedance, you will build a 50Ω input impedance into your receiving end just in case...

a. What is the effect of connecting a 50Ω output impedance device to a 50Ω input impedance device? Show how you got your result (what rule, if any, that you used.)

b. The Agilent 33120A function generator is helpfully designed with a setting for whether you are connecting it to a 50Ω impedance or to a high (say, infinite) input impedance, like a voltmeter or oscilloscope. On one of the settings, when you measure it with an oscilloscope (which has 10M input impedance), the function generator seems to put out TWICE the voltage it says it is on its display. This sometimes confuses people. Why would it do this? Which setting (for 50Ω or “High Z” input impedance) puts out 2X the voltage it says it is when you measure it with the ‘scope?
8. (15 pts) Extra credit: (§2.7.3, Photodetection & Measurement) Fixes for Resistor pathology. Resistors have "parasitic capacitance", usually due to the end-caps:

This can be modeled as a small capacitance $C$ in parallel with the resistance $R$. A typical resistor might have 0.2pF of parasitic capacitance. This can be a big deal if the resistor is used in the feedback network of the op-amp you use for your fast-pulse photodetector.

a. What is the impedance of a 100M $\parallel$ 0.2pF resistor? What is the $f_{3dB}$?

The gain of your amplifier will roll-off starting at this frequency.

$$Z_{11} = \frac{R}{J\omega C + R} = \frac{R}{1 + j\omega RC} \quad \text{so} \quad |Z_{11}| = \frac{R}{\sqrt{1 + (\omega RC)^2}}$$

$$f_{3dB} = \frac{1}{2\pi RC} = \frac{32}{32\pi \cdot 100\Omega \cdot 0.2pF} \approx 8 \text{ kHz}$$

b. Since they're built roughly the same way (similar size end-caps), the parasitic capacitance is similar for most resistors of the same type, say 0.2pF.

What if you use two 50M $\parallel$ 0.2pF resistors in series?

i. What is the impedance of the series resistors?

$$Z_{11} = \frac{50M}{\sqrt{1 + (\omega \cdot 50M \cdot 0.2pF)^2}}$$

$$Z_{series} = Z_{11} + Z_{11} = \frac{100M}{\sqrt{1 + (\omega \cdot 50M \cdot 0.2pF)^2}}$$

ii. What is the resistance at low frequency? What is the $f_{3dB}$?

As $\omega \to 0$, $Z_{series} \to 100M$ ... same as for 100M resistor.

$$f_{3dB} = \frac{1}{2\pi \cdot 50M \cdot 0.2pF} \approx 16 \text{ kHz} \quad \rightarrow \text{ twice the } 3dB \text{ for a 100M resistor!}$$

iii. What if you did this with 4 or 8 resistors in series, still all adding to 100M?

$$4R's \rightarrow f_{3dB} \approx 32 \text{ kHz}, \quad 8R's \rightarrow f_{3dB} \approx 64 \text{ kHz}$$