lculus Week 15 HW - odd

6.4: 1, 3, 5, 17, 29, 39, 57, 63 6.5: 1, 5, 9, 35

6.4 #1 $y = \ln(2x) \quad \frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$

3) $y = (\ln x)^{-1} \quad y' = -(\ln x)^{-2} \cdot \frac{1}{x}$

5) $y = x\ln x - x \quad y' = \ln x + \frac{x}{x} - 1 = \ln x$

17) $$\int_2^e \frac{dx}{x\ln x} = \int_{x=2}^{x=e} \frac{1}{u}\,du = \ln u\Big]_{u=\ln 2}^{u=\ln e = 1} = \ln 1 - \ln(\ln 2) = 0 - \ln(\ln 2)$$

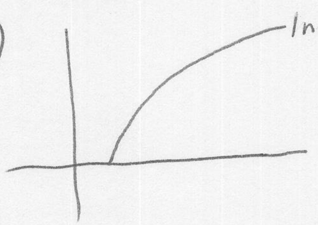
$u = \ln x$

$du = \frac{1}{x}dx$

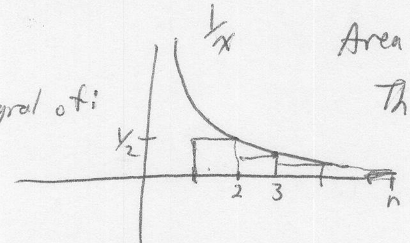
29) $y = e^{\sin x} \quad (y' = e^{\sin x} \cdot \cos x)$ ← This uses our previous method

New method: $\ln y = \sin x \Rightarrow \frac{1}{y}y' = \cos x \Rightarrow y' = y\cos x = e^{\sin x}\cos x$

39) $\frac{d}{dx}\int_x^1 \frac{dt}{t} = -\frac{d}{dx}\int_1^x \frac{dt}{t} = -\frac{1}{x}$ By FTC

57) 

is the integral of:



Area of rectangles $= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}$

This is an approximation of

$$\int_1^n \frac{1}{x}\,dx = \ln x\Big]_1^n = \ln(n) - 0$$

From diagram, we see $\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} < \ln(n)$

63) $$\lim_{x\to\infty} \frac{\ln x}{\log_{10} x} = \lim_{x\to\infty} \frac{\ln x}{\left(\frac{\ln x}{\ln 10}\right)} = \lim_{x\to\infty} \frac{\ln x}{\ln x} \cdot \ln 10 = \ln 10$$

(uses change of base formula)

6.5

1) $\frac{dy}{dt} = y + 5,\ y_0 = 2 \Rightarrow \frac{1}{y+5}dy = dt \Rightarrow \int \frac{1}{y+5}\,dy = \int dt \Rightarrow \ln|y+5| = t + c \Rightarrow e^{t+c} = y + 5$

$\Rightarrow y(t) = e^t \cdot e^c - 5$ and $y_0 = 2 = e^c - 5$

so $y_0 + 5 = e^c$

$\Rightarrow y(t) = (y_0 + 5)e^t - 5 = \boxed{7e^t - 5}$

5) $\frac{dy}{dx} = \frac{(y+1)}{x+1},\ y_0 = 0 \Rightarrow \int \frac{1}{y+1}\,dy = \int \frac{1}{x+1}\,dx \Rightarrow \ln|y+1| = \ln|x+1| + c \Rightarrow (x+1)e^c = y+1$ and $y_0 = 0 = e^c - 1$

$\Rightarrow c = 0$

$\boxed{\text{So } y = x}$

9) $\frac{dy}{dt} = C y^{1/2} \Rightarrow \int y^{-1/2}\,dy = \int C\,dt \Rightarrow 2y^{1/2} = Ct + C_0 \Rightarrow y = [\frac{1}{2}(Ct + C_0)]^2 \Rightarrow y = \frac{1}{4}(Ct + C_0)^2$

and $y_0 = y(0) \Rightarrow y(t) = \frac{1}{4}(Ct + 2\sqrt{y_0})^2$

35) $y' = y + t,\ y_0 = 0$

Let $y = Ae^t + B + Dt$

Then $y' = y + t \Rightarrow Ae^t + D = Ae^t + B + Dt + t$ (equal) Now set coefficients of like terms equal, so:

$\Rightarrow D = B$ and $(D+1) = 0$ so $y = Ae^t - 1 - t$ and since $y_0 = 0$, we have $\boxed{y = e^t - 1 - t}$