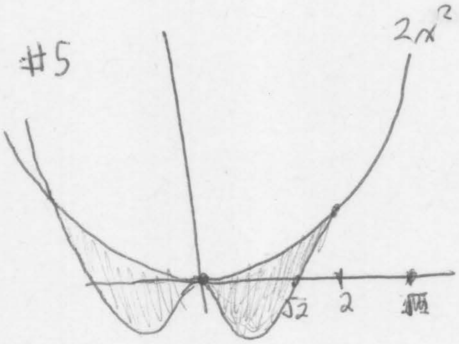
Week 18 solutions - odd

8.1 # 5, 9, 15, 29, 33.   8.2 # 7, 13, 19, 21, 29   8.3 # 1, 3, 7, 19, 21

#5



Curves intersect when $x^4 - 2x^2 = 2x^2$ so $x^4 = 4x^2$

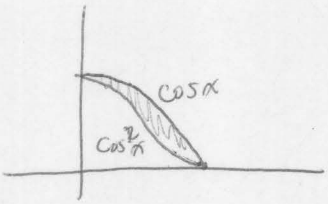
so $x = 0$ or $x = \pm 2$

Area of one lobe is $\int_0^2 2x^2 - (x^4 - 2x^2) = \int_0^2 4x^2 - x^4\,dx$

$= \left.\frac{4x^3}{3} - \frac{x^5}{5}\right]_0^2 = \frac{32}{3} - \frac{32}{5} = \frac{160-96}{15} = \frac{64}{15}$

Area both lobes is $2 \cdot \frac{64}{15} = \frac{128}{15}$

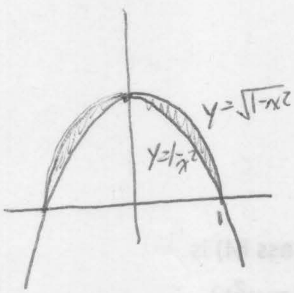
9)



$A = \int_0^{\pi/2} \cos x - \cos^2 x\,dx = \int_0^{\pi/2} \cos x - \frac{\cos 2x + 1}{2}\,dx = \left.\sin x - \frac{\sin 2x}{4} - \frac{x}{2}\right]_0^{\pi/2}$

$= 1 - \frac{\pi}{4}$

15)



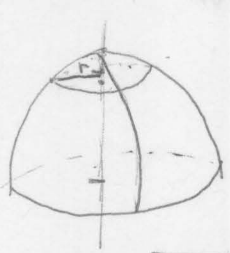
$A = 2\int_0^1 \sqrt{1-x^2} - (1-x^2)\,dx = 2\int_0^{\pi/2} \sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta - 2\int_0^1 1 - x^2\,dx$

$x = \sin\theta$
$dx = \cos\theta\,d\theta$

$= 2\int_0^{\pi/2} \cos^2\theta\,d\theta - 2\left(x - \frac{x^3}{3}\right]_0^1 = 2\int_0^{\pi/2} \frac{\cos 2\theta + 1}{2}\,d\theta - \frac{4}{3}$

$= \left.\frac{\sin 2\theta}{2} + \theta\right]_0^{\pi/2} - \frac{4}{3} = \frac{\pi}{2} - \frac{4}{3}$

29)



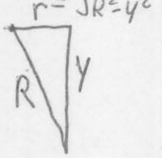
$\int_{R-h}^{R} \pi r^2\,dy = \int_{R-h}^{R} \pi(R^2 - y^2)\,dy = \left.\pi R^2 - \pi y^2\right]_{R-h}^{R} = \left.\pi R^2 y - \frac{\pi y^3}{3}\right]_{R-h}^{R} = \pi R^3 - \frac{\pi R^3}{3} - \pi R^2(R-h) + \frac{\pi(R-h)^3}{3}$

$= \frac{2\pi R^3}{3} - \frac{3\pi R^2(R-h) + \pi(R-h)^3}{3}$

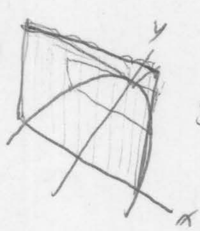
$= \frac{2\pi R^3 - 3\pi R^3 + 3\pi R^2 h + \pi R^3 - 3\pi R^2 h + 3\pi R h^2 - \pi h^3}{3}$

$= \pi R h^2 - \frac{\pi h^3}{3}$

$r = \sqrt{R^2 - y^2}$



33)



slice:

vol $= 4x^2\,dy$

$y = 1 - x^2 \Rightarrow x^2 = 1 - y$

$\text{Vol} = \int_0^1 4x^2\,dy = \int_0^1 4 - 4y\,dy = \left.4y - \frac{4y^2}{2}\right]_0^1 = 4 - \frac{4}{2} = 2$

8.2 # 7, 13, 19, 21, 29

7) $y = \frac{2}{3}x^{3/2} - \frac{1}{2}x^{1/2}$

$\frac{dy}{dx} = x^{1/2} - \frac{1}{4\sqrt{x}}$

$$\text{Length} = \int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^4 \sqrt{1+\left(x^{1/2}-\frac{1}{4\sqrt{x}}\right)^2}\,dx$$

$$= \int_1^4 \sqrt{1+x-\frac{1}{2}+\frac{1}{16x}}\,dx = \int_1^4 \sqrt{x+\frac{1}{2}+\frac{1}{16x}} = \int_1^4 \sqrt{\left(\sqrt{x}+\frac{1}{4\sqrt{x}}\right)^2}$$

$$= \int_1^4 \sqrt{x}+\frac{1}{4\sqrt{x}}\,dx = \frac{2}{3}x^{3/2} + \frac{x^{1/2}}{2}\Big]_1^4 = \frac{16}{3}+1-\frac{2}{3}-\frac{1}{2} = \frac{14}{3}+\frac{1}{2}$$

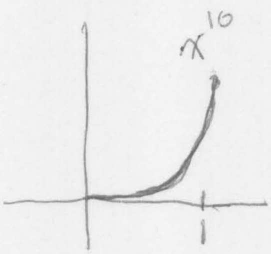
$$= \frac{31}{6}$$

13) $x = \frac{1}{2}t^2,\ y = \frac{1}{3}(2t+1)^{3/2}$

$$\text{distance} = \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt = \int_0^1 \sqrt{t^2+\left(\frac{1}{2}(2t+1)^{1/2}\right)^2} = \int_0^1 \sqrt{t^2+\frac{2t+1}{4}}\,dt = \int_0^1 \sqrt{\left(t+\frac{1}{2}\right)^2}$$

$$= \int_0^1 t+\frac{1}{2}\,dt = \frac{t^2}{2}+\frac{t}{2}\Big]_0^1 = 1$$

19)



As $n \to \infty$, x^n gets closer to the x-axis, then jumps up almost vertically to 1. So the limit of the length is

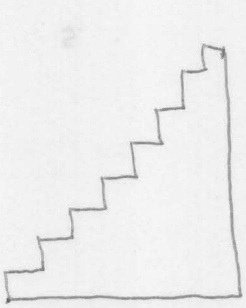
$1+1=2$

21) $x = 2\cos 3t$
$y = 2\sin 3t$

$\frac{dx}{dt} = -6\sin 3t \quad \frac{dy}{dt} = 6\cos 3t$

Speed is $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2} = \sqrt{36\sin^2 3t + 36\cos^2 3t} = 6$

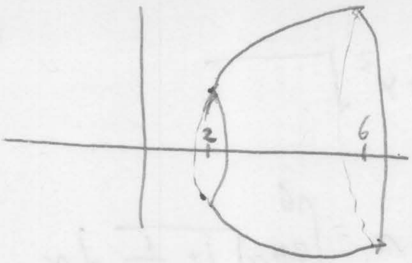
29)



Observe that for any n stairs from (0,0) to (1,1) the carpet will be of length $1+1=2$ (total vertical + total horizontal) even though the diagonal is $\sqrt{2}$.

8.3 #1, 3, 7, 19, 21

1)



$y = \sqrt{x}, \quad 2 \le x \le 6$

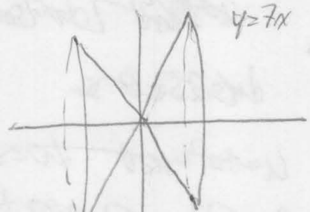
$\frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

$$SA = \int_2^6 2\pi y\sqrt{1+\frac{1}{4x}}\,dx = \int_2^6 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = 2\pi\int_2^6 \sqrt{x+\tfrac{1}{4}}\,dx$$

$$= 2\pi\left(x+\tfrac{1}{4}\right)^{3/2}\cdot\tfrac{2}{3}\Big] = \frac{4\pi}{3}\left(x+\frac{1}{4}\right)^{3/2}\Big]_2^6 = \frac{4\pi}{3}\left(\left(\frac{25}{4}\right)^{3/2} - \left(\frac{9}{4}\right)^{3/2}\right)$$

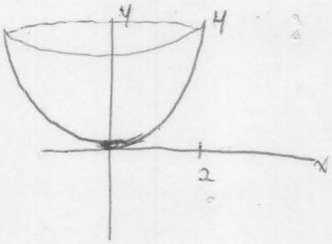
$$= \frac{4\pi}{3}\left(\frac{125}{8} - \frac{27}{8}\right) = \frac{49\pi}{3}$$

3)



$$SA = 2\int_0^1 2\pi y\sqrt{1+7^2}\,dx = 4\int_0^1 \pi 7x\sqrt{50} = 28\pi\sqrt{50}\cdot\frac{x^2}{2}\Big]_0^1 = \frac{140\pi}{\sqrt{2}}$$

7) $y = x^2 \quad 0 \le x \le 2$

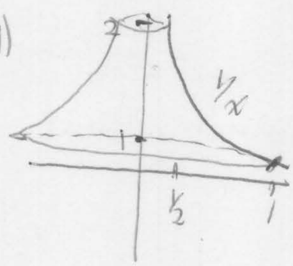


$$SA = \int_0^2 2\pi x\sqrt{1+4x^2}\,dx = \int_1^{17} \frac{\pi}{4}\sqrt{u}\,du = \frac{2u^{3/2}}{3}\cdot\frac{\pi}{4}\Big]_1^{17} = \frac{\pi}{6}\left(17^{3/2}-1\right)$$

$u = 1+4x^2$

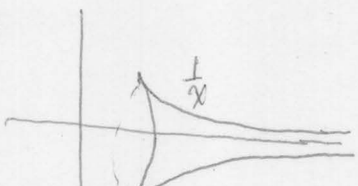
$du = 8x\,dx$

19)



$$SA = \int_{1/2}^{1} 2\pi x\sqrt{1+\frac{1}{x^4}}\,dx$$

21)



$$SA = \int_1^\infty 2\pi y\sqrt{1+\frac{1}{x^4}}\,dx = \int_1^\infty \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}\,dx$$ diverges since $\int_1^\infty \frac{2\pi}{x}$ diverges

and $\frac{2\pi}{x} < \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}$

$$Vol = \int_1^\infty \pi r^2\,dx = \int_1^\infty \pi y^2\,dx = \int_1^\infty \frac{\pi}{x^2}\,dx = \frac{-\pi}{x}\Big]_1^\infty = 0+\pi \quad \text{converges}$$

This paradox is a result of cubic units being different from square units. In this case we couldn't paint the surface with any finite thickness of paint, due to our finite volume altogether. But we could "paint" the surface with an infinitely thin layer (which allows the paint to extend to infinity).