Calculus Week 17 HW solns - odd

7.4: 3, 5, 27 7.5 5, 9, 19, 23, 37

(3) $\dfrac{1}{(x-3)(x-2)} = \dfrac{A}{x-3} + \dfrac{B}{x-2}$ $\Rightarrow 1 = (x-2)A + (x-3)B = (A+B)x - 2A - 3B$

$\Rightarrow \left.\begin{array}{l} A+B=0 \\ 2A+3B=-1 \end{array}\right\} \Rightarrow B=-1,\ A=1$

(5) $\dfrac{x^2+1}{x(x+1)(x+2)} = \dfrac{A}{x} + \dfrac{B}{x+1} + \dfrac{C}{x+2}$ "cover-up" method:

(a) $\dfrac{x^2+1}{(x+1)(x+2)} = A + \dfrac{Bx}{x+1} + \dfrac{Cx}{x+2}$. Let $x=0$, calculate $A=\frac{1}{2}$

(b) $\dfrac{x^2+1}{x\quad(x+2)} = \dfrac{A(x+1)}{x} + B + \dfrac{C(x+1)}{x+2}$. let $x=-1$, get $B=-2$

(c) $\dfrac{x^2+1}{x(x+1)} = \dfrac{A(x+2)}{x} + \dfrac{B(x+2)}{x+1} + C$. let $x=-2$, get $C=\frac{5}{2}$

(27) $\displaystyle\int \frac{dx}{1+\sqrt{x+1}} = \int \frac{2u}{1+u}\,du = \int 2 + \frac{-2}{1+u}\,du = 2u - 2\ln|1+u| + C = 2\sqrt{x+1} - 2\ln|1+\sqrt{x+1}| + C$

$u=\sqrt{x+1}$

$du = \frac{1}{2}(x+1)^{-1/2}\,dx \Rightarrow dx = 2u\,du$

$$\begin{array}{r|l} & 2 \\ u+1 & 2u \\ & 2u+2 \\ & -2 \end{array}$$

7.5

(5) $\displaystyle\int_{-\infty}^{0} \frac{dx}{x^2+1} = \tan^{-1}x\Big]_{-\infty}^{0} = \tan^{-1}0 - \lim_{x\to-\infty}\tan^{-1}x = 0 - \left(-\tfrac{\pi}{2}\right) = \tfrac{\pi}{2}$

(9) $\displaystyle\int_0^e \ln x\,dx = x\ln x\Big]_0^e - \int_0^e dx = e\ln e - \lim_{x\to0^+} x\ln x - (e-0) = e - \lim_{x\to0^+} x\ln x - e = \lim_{x\to0^+} x\ln x$

$u=\ln x \quad dv = dx$

$du = \frac{1}{x}dx \quad v = x$

L'Hopital's: $\displaystyle\lim_{x\to0^+}\frac{\ln x}{\frac{1}{x}} = \lim_{x\to0^+}\frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to0^+} -x = 0$ so $\displaystyle\int_0^e \ln x\,dx = 0$

Cool!

(19) $\displaystyle\int_0^\infty \frac{\sqrt{x}}{x^2+1}\,dx$ This can be compared with $\displaystyle\int_1^\infty \frac{\sqrt{x}}{x^2}\,dx = \int_1^\infty \frac{1}{x^{3/2}}\,dx$ which converges since $\frac{3}{2}>1$.

Also, $\displaystyle\int_0^1 \frac{\sqrt{x}}{x^2+1}\,dx$ is finite, so converges. Thus $\displaystyle\int_0^\infty \frac{\sqrt{x}}{x^2+1}\,dx = \underbrace{\int_0^1 \frac{\sqrt{x}}{x^2+1}\,dx}_{\text{finite}} + \underbrace{\int_1^\infty \frac{\sqrt{x}}{x^2+1}\,dx}_{\text{converges since } \frac{\sqrt{x}}{x^2+1} < \frac{\sqrt{x}}{x^2}}$ converges

(23) $\displaystyle\int_0^\infty e^{2x}e^{-x^2}\,dx = \int_0^\infty \frac{1}{e^{x(x-2)}}\,dx = \int_0^3 \frac{1}{e^{x(x-2)}}\,dx + \int_3^\infty \frac{1}{e^{x(x-2)}}\,dx$

But when $x\geq2$ we have $\frac{1}{e^{x(x-2)}} < \frac{1}{e^x}$

So $\displaystyle\int_3^\infty \frac{1}{e^{x(x-2)}}\,dx < \int_3^\infty \frac{1}{e^x} = -e^{-x}\Big]_3^\infty = 0 + \frac{1}{e^3} = \frac{1}{e^3}$ converges. Hence $\displaystyle\int_0^\infty \frac{1}{e^{x(x-2)}}$ also converges.

(37) $\int_0^{\pi/2} \sec x - \tan x \, dx = \int_0^{\pi/2} \frac{1-\sin x}{\cos x} dx = \int_0^{\pi/2} \frac{(1-\sin x)(1+\sin x)}{\cos x\,(1+\sin x)} dx = \int_0^{\pi/2} \frac{1-\sin^2 x}{\cos x\,(1+\sin x)} dx$

$= \int_0^{\pi/2} \frac{\cos^2 x}{\cos x(1+\sin x)} dx = \int_0^{\pi/2} \frac{\cos x}{1+\sin x} dx = \int_1^2 \frac{du}{u} = \ln u \Big]_1^2 = \ln 2$

$u = \sin x + 1$
$du = \cos x \, dx$

(Notice that both $\sec x$ and $\tan x \to \infty$ as $x \to \pi/2$ making each term in the difference $\sec x - \tan x$ improper in the interval from 0 to $\pi/2$. However the differen itself does not approach ∞)