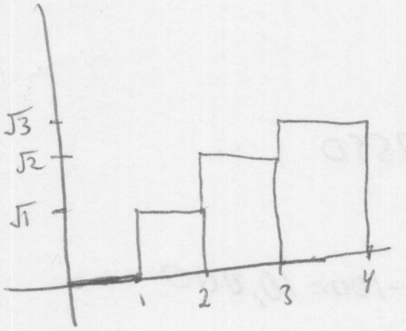
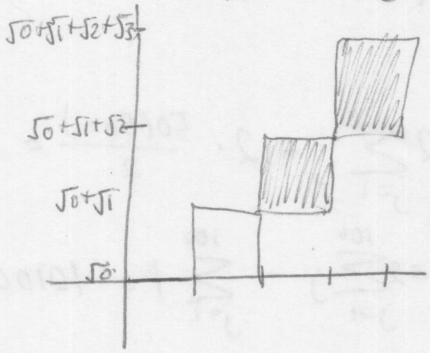
Calculus Week 11 - Solns. to odd problems

5.1: 11, 23 5.2: 3, 7, 25, 27 5.3: 1, 5, 7, 13, 29, 35

11)





This estimate of total income is lower than that in Fig 5.1 by $\sqrt{4}$

23) $\Delta x = \frac{1}{3}$

$$\text{Area} = \left(\frac{1}{3}\right)^2 \cdot \frac{1}{3} + \left(\frac{2}{3}\right)^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{1}{3}$$
$$= \frac{14}{27}$$

$\boxed{5.2}$ 1) Antiderivative of $5x^4+4x^3$ is x^5+x^4 so integral $\int_0^1 5x^4+4x^3\,dx$ is $x^5+x^4\Big]_0^1 = 2$

3) An antiderivative of $\frac{1}{\sqrt{x}} = x^{-1/2}$ is $2x^{1/2}$, so $\int_0^1 \frac{1}{\sqrt{x}}\,dx = 2\cdot 1^{1/2} - 2\cdot 0^{1/2} = 2$

7) An antiderivative of $2\sin x + \sin 2x = -2\cos x - \frac{\cos 2x}{2}$

So $\int_0^1 2\sin x + \sin 2x\,dx = -2\cos x - \frac{\cos 2x}{2}\Big]_0^1 = -2\cos 1 - \frac{\cos 2}{2} + 2\cos 0 + \frac{\cos 0}{2}$

$= -2\cos 1 - \frac{\cos 2}{2} + \frac{5}{2}$

25) In the first graph we see about area 1 above the axis and $\frac{1}{2}$ below.
So $f(2) \approx 1 - \frac{1}{2} = \frac{1}{2}$. For $f(4)$ note area below ≈ 2 so $f(4) \approx 1-4 = -3$

In the second figure, area above is $\frac{1}{2}$ and area below is $\frac{1}{4}$ from 1 to 2. Hence $f(2) \approx \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

Going on to 4 we add positive area of 1, so $f(4) \approx 1\frac{1}{4} = \frac{5}{4}$

27) For v_1 the area increases then decreases after 1

For v_2 the area increases then decreases from 1 to 2 then increases from 2 to 4.

1) $\sum_{n=1}^{4} \frac{1}{n} = 1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4} = \frac{25}{12}$ $\quad \sum_{i=2}^{5} (2i-3) = 1+3+5+7=16$

5) $2+4+6+\dots+100 = \sum_{j=1}^{50} 2j = 2\sum_{j=1}^{50} j = 2\cdot\frac{50(50+1)}{2} = 2550$

$1+3+5+\dots+199 = \sum_{j=1}^{100} 2j-1 = 2\sum_{j=1}^{100} j - \sum_{j=1}^{100} 1 = 10100-100 = 10,000$

$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4} = \sum_{j=1}^{4} (-1)^{j+1}\cdot\frac{1}{j} = \frac{7}{12}$

7) $a_0+\dots+a_nx^n = \sum_{j=0}^{n} a_n x^n$

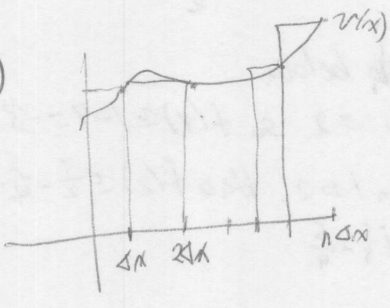
$\sin\frac{2\pi}{n}+\sin\frac{4\pi}{n}+\dots+\sin 2\pi = \sum_{j=1}^{n} \frac{2\pi j}{n}$

13) $\sum_{k=1}^{n} 2^k-2^{k-1} = \cancel{2^1}-2^0+\cancel{2^2}-\cancel{2^1}+2^3-\cancel{2^2}+\dots+2^n-\cancel{2^{n-1}} = 2^n-1$

$\sum_{j=1}^{10} \frac{1}{j+1}-\frac{1}{j} = \cancel{\frac{1}{2}}-1+\cancel{\frac{1}{3}}-\cancel{\frac{1}{2}}+\cancel{\frac{1}{4}}-\cancel{\frac{1}{3}}+\dots+\frac{1}{11}-\cancel{\frac{1}{10}} = \frac{1}{11}-1 = \frac{-10}{11}$

29) $\sum_{i=1}^{2}\left[\sum_{j=1}^{3}(i+j)\right] = \sum_{j=1}^{3} 1+j + \sum_{j=1}^{3} 2+j = \underbrace{2+3+4}_{v_1} + \underbrace{3+4+5}_{v_2} = 9+12=21$

35)



$\text{Area} = \Delta x\, v(\Delta x) + \Delta x\, v(2\Delta x) + \Delta x\, v(3\Delta x) + \dots \Delta x\, v(n\Delta x)$

$= \sum_{j=1}^{n} v(j\Delta x)\cdot\Delta x$