Week 14 calculus Solutions - odd problems

6.1: 1, 5, 13, 15, 23, 27   6.2: 1, 3, 5, 13, 17, 27, 31, 45,   6.3: 1, 3, 9, 19

6.1

(1)(a) $\log_2 32 = 5$ since $2^5 = 32$

(b) $\log_2\left(\frac{1}{32}\right) = -5$ since $2^{-5} = 32$

(c) $\log_{32}\left(\frac{1}{32}\right) = -1$

(d) $\log_{32} 2 = \frac{1}{5}$

(e) $\log_{10}(10\sqrt{10}) = \log_{10}(10^{3/2}) = 3/2$

(f) $\log_2(\log_2 16) = \log_2(4) = 2$

(5)(a) $\log_2 3 + \log_2 \frac{2}{3} = \log_2\left(3 \cdot \frac{2}{3}\right) = \log_2 2 = 1$

(b) $\log_2\left(\frac{1}{2}\right)^{10} = 10\log_2\left(\frac{1}{2}\right) = 10 \cdot (-\log_2 2) = -10$

(c) $\log_{10} 100^{40} = \log_{10}(10^2)^{40} = \log_{10} 10^{80} = 80\log_{10} 10 = 80$

(d) $(\log_{10} e)(\log_e 10) = \frac{\log_e e}{\log_e 10} \cdot \log_e 10 = \log_e e = 1$

(e)
$$\frac{2^{2^3}}{(2^2)^3} = \frac{2^8}{2^6} = 2^2 = 4$$

(f) $\log_e\left(\frac{1}{e}\right) = -\log_e e = -1$

(13) Increase from 20db to 70db is by a factor of $A = 10^5$ since $10\log A = 50 \Rightarrow \log A = 5$

(15)
- $R = \log_{10}\left(\frac{I}{I_0}\right)$ so $R = \log_{10}\frac{I_0}{I_0} = \log_{10} 1 = 0$ for standard I_0 intensity.
- If $R = 7$ then $7 = \log_{10}\frac{I}{I_0}$ so $\frac{I}{I_0} = 10^7$ and $I = 10^7 I_0$.
- SF Quake: $8.3 = \log_{10}\frac{I}{I_0}$ Record Quake $R = \log_{10}\frac{4I}{I_0} = \log_{10} 4 + \log_{10}\frac{I}{I_0} = \log_{10} 4 + 8.3$

(23) $x = \log_{10} y = \frac{\ln y}{\ln 10}$

$\frac{dx}{dy} = \frac{1}{\ln 10} \cdot \frac{1}{y}$ At $y=1$, $\frac{dx}{dy} = \frac{1}{\ln 10} = m$. Tang. line passes through point $y=1, x=0$ and its equation is $x = \frac{1}{\ln 10} y - \frac{1}{\ln 10}$

At $y=10$, $\frac{dx}{dy} = \frac{1}{10\ln 10} = m$ Tang line through $y=10, x=1$ is $x = \frac{1}{10\ln 10} y - \frac{1}{\ln 10} + 1$

(27) $y = b^x$ $x = \log_b y$

$\Rightarrow y' = \ln b \cdot b^x$ $\Rightarrow x' = \frac{1}{y\ln b} = \frac{1}{\ln b} y^{-1}$

$\Rightarrow y'' = (\ln b)^2 b^x$ $\Rightarrow x'' = \frac{-1}{\ln b} y^{-2}$

6.2

(1) $\frac{d}{dx} 7e^{7x} = 49e^{7x}$

(3) $\frac{d}{dx}(e^x)^8 = \frac{d}{dx} e^{8x} = 8e^{8x}$

(5) $\frac{d}{dx} 3^x = \ln 3 \cdot 3^x$

(13) $\frac{d}{dx}(xe^x - e^x)$

$= \underbrace{e^x + xe^x}_{\text{product rule}} - e^x = xe^x$

(17) $\frac{d}{dx}\left(e^{\sin x} + \sin e^x\right)$

$= \cos x\, e^{\sin x} + (\cos e^x) \cdot e^x$

(27) $\int e^{3x} + e^{7x} = \frac{e^{3x}}{3} + \frac{e^{7x}}{7} + C$

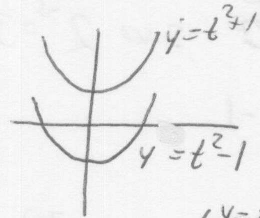
(31) $\int \left((2e)^x + 2e^x\right) dx = \dfrac{(2e)^x}{\ln(2e)} + 2e^x + C = \dfrac{(2e)^x}{\ln 2 + 1} + 2e^x + C$

(45) $\int_0^1 e^{2x}\,dx = \left.\dfrac{e^{2x}}{2}\right]_0^1 = \dfrac{e^2}{2} - \dfrac{1}{2}$

6.3

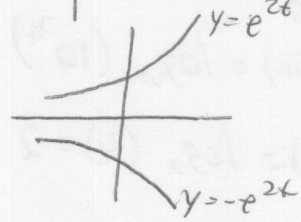
(1) $\dfrac{dy}{dt} = 2t \Rightarrow y = t^2 + C$ $\Rightarrow y = t^2+1$ if $y_0 = 1$

$\Rightarrow y = t^2 - 1$ if $y_0 = -1$

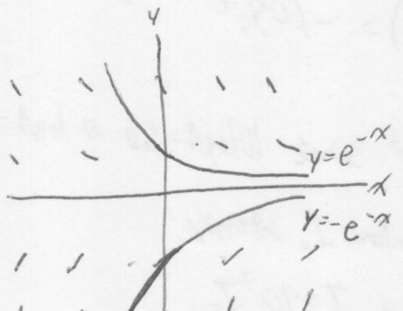


(3) $\dfrac{dy}{dt} = 2y \Rightarrow y = Ce^{2t}$ $\Rightarrow y = e^{2t}$ if $y_0 = 1$

$\Rightarrow y = -e^{2t}$ if $y_0 = -1$



(9) $y' = -y$



(19) $y = 1000$ at $t = 3$

$y = 3000$ at $t = 4$

$y = y_0 e^{ct}$ so $1000 = y_0 e^{3c}$, $3000 = y_0 e^{4c}$ $\Rightarrow 3 = \dfrac{y_0 e^{4c}}{y_0 e^{3c}} = e^c \Rightarrow c = \ln 3$

Also, $1000 = y_0 e^{(\ln 3)3} = 3^3 y_0 \Rightarrow y_0 = \dfrac{1000}{27}$