Week 13 calculus solutions - odd problems

5.7: 1, 5, 7, 11, 19, 27

5.8: 5, 7, 13, 25, 29

5.7

1) $\frac{d}{dx}\int_1^x \cos^2 t\, dt = \cos^2 x$

5) $\frac{d}{dx}\int_1^{x^2} u^3\, du = (x^2)^3 \cdot 2x = 2x^7$

7) $\frac{d}{dx}\int_x^{x+1} v(t)\, dt = \frac{d}{dx}\left(\int_x^a v(t)\, dt + \int_a^{x+1} v(t)\, dt\right) = -v(x) + v(x+1)$

11) $\frac{d}{dx}\int_0^x \left[\int_0^t v(u)\, du\right] dt = \int_0^x v(u)\, du$

19) $\frac{d}{dx}\int_0^{\sin x} \sin^{-1} t\, dt = \sin^{-1}(\sin x) \cdot \cos x + C = x\cos x$

or the value between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ which has the same sin as x does.

27) (a) False (e.g. $|x|$ is continuous but not diff'ble at $x=0$)

(b) True (all cont. fcns can be integrated)

(c) Yes, by defn an antiderivative must have a derivative.

(d) False. $\frac{d}{dx}\int_{2x}^{3x} \frac{1}{t^2}\, dt = \frac{1}{(3x)^2} \cdot 3 - \frac{1}{(2x)^2} \cdot 2 = \frac{1}{3x^2} - \frac{1}{2x^2} = \frac{-1}{6x^2} \neq 0$

5.8

5) $I = \int_0^1 x^4\, dx$ for $n = 2, 4, 8$

n=2

$L_2 = 0 + \left(\frac{1}{2}\right)^4 = \frac{1}{16} \cdot \frac{1}{2} = \frac{1}{32}$

$R_2 = \frac{1}{2}\left(\frac{1}{16} + 1\right) = \frac{17}{16} \cdot \frac{1}{2} = \frac{17}{32}$

$T_2 = \frac{1}{2}L_2 + \frac{1}{2}R_2 = \frac{18}{64} = \frac{9}{32}$

n=4

$L_4 = 0 + \left(\frac{1}{4}\right)^4 \cdot \frac{1}{4} + \left(\frac{1}{2}\right)^4 \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^4 \cdot \frac{1}{4} \approx .0957$

$R_4 \approx .3457$ Geogebra

$T_4 = \frac{1}{2}(L_4 + R_4) \approx \frac{1}{2}(.4414) = .2207$

n=8

$L_8 \approx .1427$

$R_8 \approx .2677$

$T_8 \approx \frac{1}{2}(.4104) = .2052$

7) New Simpson's Rule:

$$\Delta x\left(\tfrac14 y_0+\tfrac12 y_{1/2}+\tfrac14 y_1\right)$$

$$\int_0^1 x\,dx \approx 1\left(\tfrac14\cdot 0+\tfrac12\cdot\tfrac12+\tfrac14\cdot 1\right)=\tfrac12 \checkmark \quad \text{which is equal to} \quad \int_0^1 x\,dx=\frac{x^2}{2}\Big]_0^1=\frac12$$

However,

$$\int_0^1 x^2\,dx \approx 1\cdot\left(\tfrac14\cdot 0+\tfrac18+\tfrac14\right)=\tfrac38 \neq \int_0^1 x^2\,dx=\tfrac13 \quad \text{so} \quad p=2 \quad \text{(second order)}$$

13) Leading error is $(\Delta x)^2\cdot\frac{y'(b)-y'(a)}{12}$ and $y'(x)=-\frac{1}{x^2}$ so we want to find n such that $\left(\frac1n\right)^2\cdot\frac{\left(-\frac14+1\right)}{12}<.001$, so $\frac{3}{48}<.001\,n^2$.

That is, $\frac{3000}{48}<n^2$ so $n>\sqrt{\frac{3000}{48}}=7.9$ So use $n=8$ intervals.

In that case, $T_8\approx .692$ (Geogebra - could be inexact)

25)
$$\left.\begin{aligned}1&=c\\ 3&=a\cdot\tfrac14+b\cdot\tfrac12+1\\ 7&=a+b+1\end{aligned}\right\}\Rightarrow \left.\begin{aligned}8&=a+2b\\ 6&=a+b\end{aligned}\right\}\Rightarrow \begin{aligned}b&=2,\ a=4\\ c&=1\end{aligned}$$

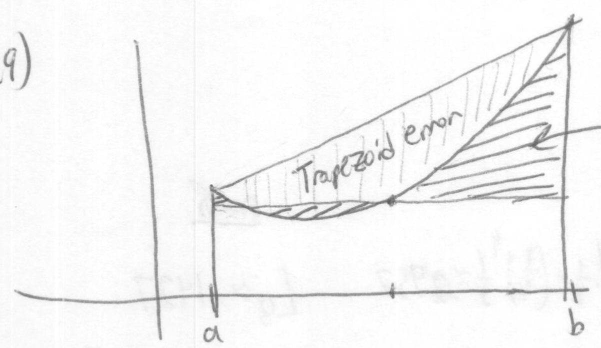
Then $y=4x^2+2x+1$

$$\int_0^1 4x^2+2x+1\,dx=\frac{4x^3}{3}+x^2+x\Big]_0^1=\frac43+2=\frac{10}{3}\overset{?}{=}\frac16+\frac{12}{6}+\frac76$$

$$\frac{10}{3}=\frac{10}{3}\ \checkmark$$

(This shows that Simpson's rule corresponds to a quadratic approximation)

29)



midpoint error (more area above rectangle than below)

(5) $T_n-I\approx\frac{1}{12}(\Delta x)^2\left[y_n'-y_0'\right]$

If $y''>0$ then y' is increasing so $y_n'-y_0'>0$

(6) $M_n-I\approx-\frac{1}{24}(\Delta x)^2\left[y'(b)-y'(a)\right]$

If $y''>0$ then $y'(b)-y'(a)>0$ so $-(y'(b)-y'(a))<0$.