Lab 21: Electric Fields – Visualizing Vectors and Numerical Integration

Goals: Visualize the electric field due to a configuration of discrete charges; Practice the math involved in the Riemann Sum; Use numerical integration to determine the electric field due to a distribution of charge.

Groups & Lab Notebook: Work by yourself. Though consultation with your classmates is encouraged, you should make sure to only be checking work that you yourself have carried out by yourself or ask clarifying questions. Update Table of Contents. Take good notes.

Part I: Visualizing Electric Field Vectors
Launch the simulation Charges and Fields 2.04 available at http://phet.colorado.edu/en/simulation/charges-and-fields by going to that site and then clicking on the green button Run Now! Set up the charges (each charge shown is a single charge) as shown in each figure below. Consider the field points represented by the empty circles and draw approximately the electric field vector at those field points. Then, use the E-field sensors to find the electric field for each of the empty circles (those are the field points) and draw those in. Finally, choose your own charge configuration, predict what the electric field will look like at various points, and test out your predictions.

Part II: Numerical Integration for Electric Fields
We use numerical integration to evaluate definite integrals when an anti-derivative is difficult or impossible to determine. The specific problem this will be applied to is the electric field of distributed charges. We begin by setting up the integral. Then, we carry out the integration via numerical methods; here we use an implementation of Riemann Sums in Desmos.

Consider the rod shown in Figure 1. This rod has a total length $L$ and a total charge $Q$ distributed uniformly along it. The rod is assumed to be thin enough that its width can be neglected. It is oriented along the $y$-axis with the center at the origin. The $x$-axis is chosen to point in the direction perpendicular from the rod to the field point $P$. This simplifies the problem allowing the z-axis and components to be ignored (due to the symmetry of the problem). The coordinates of the field point are $(x_P, y_P)$. Though this system can be treated purely analytically (as you have seen), we’ll get practice with numerical integration in this case, where we can check our answers analytically.

The rod can be split into pieces and each piece can be treated as a point charge. If the pieces are small enough, this is a good approximation and the results will be very close to the true value. However, the technique can be used even if there are not many pieces and the pieces are not small; the resulting solution will not be as good an approximation, though. In the following exercise, you will split the rod up into 3 equal parts.

Exercise #1: Consider a rod of length 1 m with charge 10 $\mu$C uniformly distributed along its length that lies along the $y$-axis, with its center at the origin, as shown in Figure 1. Split the rod into three equal pieces with the charge of each piece all located at the center of that piece. Then, determine the electric field at the point (0.4 m, 0.0 m) due to these 3 pieces. Do the calculations by hand and calculator to get a sense for what is going on.
Exercise #2: For the rod as shown in Figure 1, determine the definite integrals which would allow you to calculate the x-component and y-component of the electric field. There’s no need to evaluate these definite integrals by hand, as we’ll see next how to evaluate them numerically.

You have been provided with a Desmos shell which use Riemann Sums to approximate integrals, at https://www.desmos.com/calculator/qjtomj8cgf. This is similar to the GeoGebra applets you have worked with previously but a bit more customizable.

A note about units: computers have no concept of units. All the numerical values in Desmos have no units attached to them. This does not mean that units can be neglected when doing these calculations. Rather, even more attention must be paid to the units since there is no chance of catching errors when doing the algebra and/or arithmetic. The way this is most easily done is requiring that all values are expressed in standard units: meters for length, Coulombs for charge, Newtons per Coulombs for electric field, etc.

Examine the Desmos shell you have been provided. Note that a function has already been input into line 2 (this function is close to but not necessarily identical to any you derived previously). Scroll down to see the constants and inputs, any of which can be adjusted (some have sliders, but ignore those and just type in any values you need). You can see places to enter the geometrical endpoints which are also the limits of integration. You can see how to adjust the number of pieces (or partitions or intervals…). You can see the Area function, which calculates the area (note that the output here gives a ridiculous number of digits). Compare the Area function to your integral expressions.

Optional for after you complete this lab. Those of you who are interested can click on the Formulas folder and the Curves and coloring Folder to see the details of how all this is implemented in Desmos.

Exercise #3: Modify the provided Desmos shell to numerically determine the values for the definite integrals you found in Exercise #2. Enter formulas into Desmos that will calculate the electric field (actually the x- and y-components of the electric field) anywhere in the x-y plane for a finite rod of fixed uniform charge. You can either do this twice in a row with different formulas each time, or you can figure out how to calculate both components at the same time (note that your graphs will get a bit confusing in that case).

a) Use Desmos to duplicate your results from Exercise #1 by entering in the formula for one of your definite integrals (note that you shouldn’t have k, Q, or L in this formula since those come in separately in the Area function) into line 2 and also setting N = 3.

b) Now, split the rod into 20 equal pieces. In Desmos, this should be easy to do; just change the value for N as needed. Verify that it works by increasing N slowly and examining the graph.

c) Use Desmos to calculate the electric field at the point (0.3 m, 0.4 m), still for 20 equal pieces. This should be easily accomplished just by changing the values for \(x_P\) and \(y_P\). You can check that you’ve done this correctly by comparing to my results, which have the x-component as 3.798 \(\times\) \(10^5\) (N/C) and the y-component as 1.900 \(\times\) \(10^5\) (N/C).

d) Use Desmos to calculate the electric field at the point (–1.0 m, 2.0 m). Comment on your result.

e) Investigate the effects of increasing \(N\). Change from 10 to 100 to 1000 to 10000, etc.

Exercise #4: Find the integrals that will allow you to calculate the components of the electric field due to a uniform ring of charge, and implement in Desmos. The Desmos equations should be able to calculate the electric field anywhere in the plane of the ring. Even though this ring seems two-dimensional, you still only need one counting variable (hint: think about the unit circle, and see class notes and Example 23.5).

Consider as your source a uniformly charged ring (of negligible thickness) that lies in the x-y plane with its center at the origin. The ring has radius 0.10 m and total charge 0.005 C. You should break up the ring into 100 evenly sized pieces.

a) Write down integral expressions for the x-component and y-component of the electric field due to a generic ring with charge \(Q\) and radius \(a\) anywhere in the plane of the ring. Can you find anti-derivatives for these expressions?

b) Implement your expression(s) in Desmos. Confirm your work by checking the following cases:

- The field at the center of the ring
- The direction of the electric field at \((0, –0.20 \text{ m})\)
- The field at a very far distance on the x-axis, say (20.0 m, 0) (note: very far from the ring, can treat as point).

c) Report the electric field at (0.20 m, 0.30 m). Comment on your result.

Challenge Exercise #5: What if the charge is not uniformly distributed? Consider the ring from Exercise #4, but this time, have the charge density vary as a function of angle, so that \(\lambda = \lambda_0 \sin \theta\), where \(\lambda_0 = 0.001 \text{ C/m}\). How does your integral change? How do you implement this in your numerical integration procedure?