Calculus HW9 solutions to odd problems

4.3: 1, 5, 13, 23, 25, 29, 33, 45

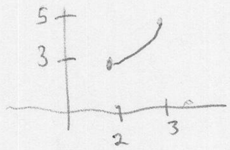
4.4: 1, 5, 15, 19, 27, 33, 39

4.3

1) $y=3x-6 \Rightarrow y+6=3x \Rightarrow x=\frac{y+6}{3}=g^{-1}(y)$

5) $y=1+\frac{1}{x} \Rightarrow y-1=\frac{1}{x} \Rightarrow x=\frac{1}{y-1} \quad (y\neq 1)$

13) $f(2)=3,\ f(3)=5$ Since f is increasing, $f^{-1}(4)$ must be between 2 and 3



23) $y=x^3-1 \qquad \frac{dy}{dx}=3x^2 \qquad \frac{dx}{dy}=\frac{1}{3x^2}=\frac{1}{3(y+1)^{2/3}}$

$\Rightarrow x=\sqrt[3]{y+1}$

25) $y=\frac{x}{x-1} \qquad \frac{dy}{dx}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2} \qquad \frac{dx}{dy}=-(x-1)^2=-\left(\frac{y}{y-1}-1\right)^2=-\left(\frac{1}{(y-1)^2}\right)$

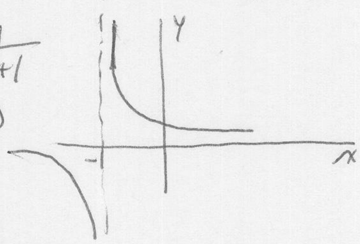
$\Rightarrow xy-y=x$

$\Rightarrow x=\frac{y}{y-1}$

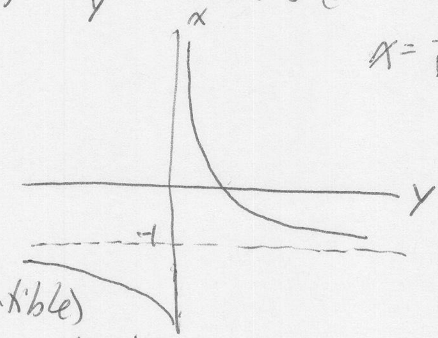
29) $f\neq g^{-1}$ since although $f'=\frac{1}{(g^{-1})'}$, it is <u>not</u> the case that $f'(x)=\frac{1}{[g^{-1}(y)]'}$ (note variable change)

In fact, ~~$g^{-1}(y)=\sqrt[3]{3v}$~~ $g^{-1}(y)=\frac{-1}{y}$ and hence $g'(g^{-1})'=\frac{1}{x^2}\cdot\frac{1}{y^2}=\frac{1}{x^2}\cdot\frac{1}{(\frac{1}{x})^2}=\frac{1}{1}=1$

33) $y=\frac{1}{x+1}$ $=g(x)$



$x=\frac{1-y}{y}=g^{-1}(y)$ (Note axes shift)

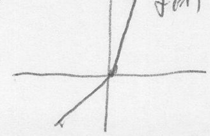


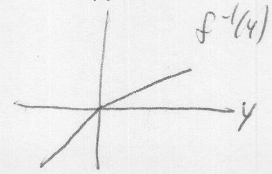
45) (a) False e.g. $f(x)=x$ (invertible)

$h(x)=x^2$ (not invertible)

(b) True $h^{-1}(x)=f^{-1}(f^{-1}(x))$ e.g. $f(x)=x+1 \quad f(f(x))=h(x)=x+2$ and $h^{-1}(x)=x-2$

(c) False $f(x)=\begin{cases} x & x\leq 0 \\ 2x & x>0 \end{cases} \Rightarrow f^{-1}(y)=\begin{cases} y & y\leq 0 \\ \frac{y}{2} & y>0 \end{cases}$





f and f^{-1} have no derivatives at 0.

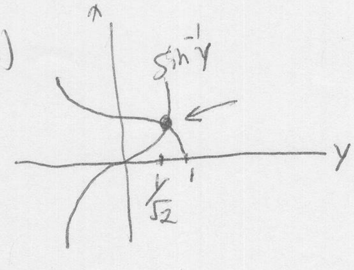
4.4

1) $\sin^{-1} 0 = 0 \quad \cos^{-1} 0 = \pi/2 \quad \tan^{-1} 0 = 0$

5) Because we restrict domain of sine to just $[-\pi/2, \pi/2]$ when finding inverse.
Hence $\sin^{-1} 0 = 0$

15) (a) $(\sin^{-1} y)^2 + (\cos^{-1} y)^2 = 1$ False. E.g. if $y=0$, $\sin^{-1} y = 0$, $\cos^{-1} y = \pi/2$ but $0 + \left(\frac{\pi}{2}\right)^2 \neq 1$

(b) False $\sin^{-1} y = \cos^{-1} y \Rightarrow y = \frac{1}{\sqrt{2}}$

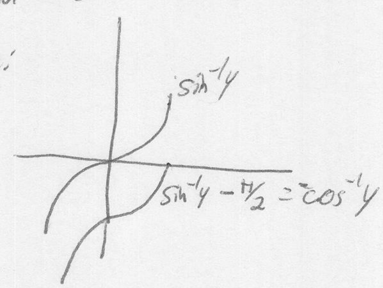


(c) true (see graph in (b))

(d) true, since $\sin^{-1}(-y) = a \Rightarrow -y = \sin a \Rightarrow y = -\sin a \Rightarrow y = \sin(-a) \Rightarrow -a = \sin^{-1} y$
$\Rightarrow a = -\sin^{-1} y$

Thus $\sin^{-1}(-y) = -\sin^{-1}(y)$ so $\sin^{-1}$ is odd.

(e) False. Having the same slope doesn't mean the functions are the same. They can differ by a constant, as is true here:



(f) False. e.g. if $x=0$, $\sin(\cos 0)) = \sin 1$
but $\cos(\sin 0) = \cos 0 = 1 \neq \sin 1$

19) $z = \sin^{-1}(\sin 3x)$

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-(\sin 3x)^2}} \cdot \cos 3x \cdot 3 = \frac{3\cos 3x}{\sqrt{\cos^2 3x}} = 3 \cdot 1 = 3$$

27) $u = \dfrac{\sin^{-1} y}{\cos^{-1}\sqrt{1-y^2}}$ Quotient $\Rightarrow$ Rule

$$\frac{du}{dy} = \frac{\cos^{-1}\left(\sqrt{1-y^2}\right) \cdot \frac{1}{\sqrt{1-y^2}} - \sin^{-1}(y) \cdot \frac{1}{\sqrt{1-(1-y^2)}} \cdot \frac{1}{2}(1-y^2)^{-1/2}(-2y)}{\left(\cos^{-1}\sqrt{1-y^2}\right)^2} = 0$$

or $\xrightarrow{\text{shortcut}}$ (observe $\sin^{-1} y = \cos^{-1}\left(\sqrt{1-y^2}\right)$) $\Rightarrow u = 1 \Rightarrow \frac{du}{dy} = 0$

33) $x = \tan^{-1} y \Rightarrow \frac{dx}{dy} = \frac{1}{1+y^2}$ (a) $\frac{1}{1+9} = \frac{1}{10}$ (b) $\frac{1}{1+(\tan^{-1} 0)^2} = 1$ (c) $\frac{1}{1+[\tan(-\frac{\pi}{4})]^2} = \frac{1}{2}$

39) $u = \sec^{-1}(x^n)$

$$\frac{du}{dx} = \frac{1}{|x^n|\sqrt{x^{2n}-1}} \cdot n x^{n-1}$$