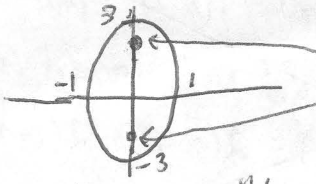
Calculus HW week 7 solutions (odd)

3.5: 9, 19, 21, 31 3.2: 61, 65, 73

3.5

(9) $9x^2+y^2=9$



foci $(0, 2\sqrt{2})$ and $(0,-2\sqrt{2})$

$x^2+\frac{y^2}{9}=1$ $\Rightarrow$ ellipse with foci $y=\pm\sqrt{b^2-a^2} = \pm\sqrt{9-1} = \pm\sqrt{8} = \pm 2\sqrt{2}$

(19) (a) $x = ky^2$ $k>0$

(b) $y=ax^2+bx+c$ focus $=\frac{1}{4a}$ above vertex and vertex $=\left(\frac{-b}{2a}, \frac{ab^2}{4a^2}-\frac{b^2}{2a}+c\right)$

Hence $\frac{-1}{4a} = \frac{b^2}{4a}-\frac{b^2}{2a}+c = c-\frac{b^2}{4a}$. But since $\frac{-b}{2a}=0$ we have $b=0$. $\therefore$ Vertex must be at $(0,c)=(0,\frac{-1}{4a})$ so $c=\frac{-1}{4a}$

So parabola is $y=ax^2-\frac{1}{4a}$

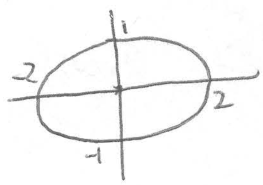
(c) $y=ax^2+bx+c$ with $a<0$

$(0,0)$: $0=0+0+c \Rightarrow c=0$

$(1,0)$: $0=a+b \Rightarrow -a=b$

So parabola is $y=ax^2-ax$ with $a<0$

(21)



This ellipse has $a=2$, $b=1$

So $\frac{x^2}{4}+y^2=1$

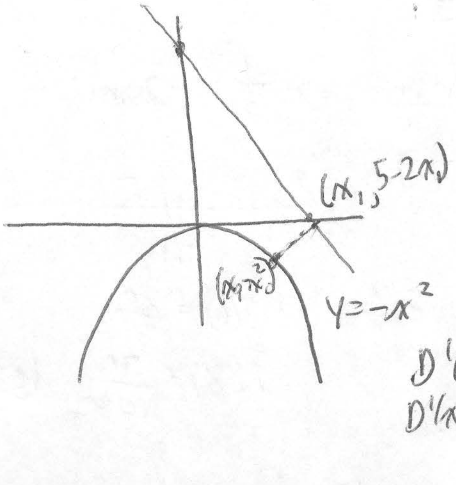
shift center to $(1,1)$: $\frac{(x-1)^2}{4}+(y-1)^2=1$

(31) This ellipse is really a circle with center at $(3,1)$ and radius $=2$.

If $X=\frac{x-3}{2}$ and $Y=\frac{y-1}{2}$ then the equation is $X^2+Y^2=1$.

3.2

#61



Distance between $(x_0,-x_0^2)$ and $(x_1, 5-2x_1)$ is

$d(x_0,x_1)=\sqrt{(x_0-x_1)^2+(-x_0^2-(5-2x_1))^2}$. We can reduce this to one variable by noting slopes need to be the same,

so $-2x_0=-2$ hence $x_0=1$. Thus:

$d(x_1)=\sqrt{(1-x_1)^2+(-1-(5-2x_1))^2}$. It's easiest to minimize

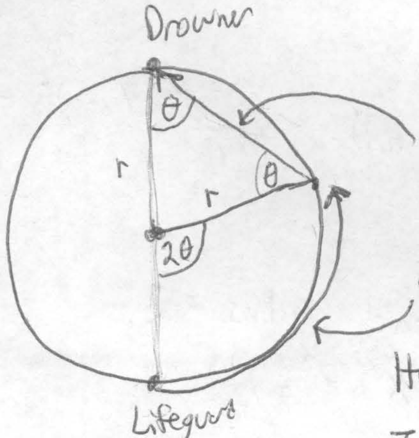
$y=-x^2$ the square of distance: $D(x)=(1-x)^2+(-6+2x)^2$.

$D'(x)=-2(1-x)+2(-6+2x)(+2)=-2+2x-24+8x=10x-26$

$D'(x)=0$ when $x=\frac{13}{5}$. Thus $(1,-1)$ and $\left(\frac{13}{5},\frac{-1}{5}\right)$ are closest

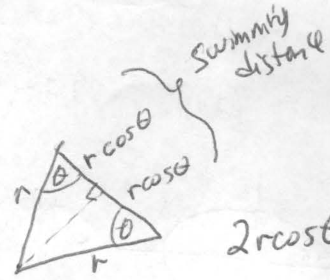
(parabola) (line)

(73)



r = radius circle

Distance the lifeguard swims is:



$2r\cos\theta$

Distance the lifeguard runs is: $2r\theta$

Hence if T = arrival time we have

$$T(\theta) = \frac{\text{Distance run}}{\text{velocity running}} + \frac{\text{Distance swum}}{\text{vel. swimming}}$$

$$= \frac{2r\theta}{10v} + \frac{2r\cos\theta}{v}$$

$$T'(\theta) = \frac{r}{5v} - \frac{2r\sin\theta}{v} = 0 \quad \text{when} \quad \frac{r}{5v} = \frac{-2r\sin\theta}{v} \Rightarrow \frac{1}{5} = -2\sin\theta$$

$$\Rightarrow \sin\theta = \frac{-1}{10} \Rightarrow \theta = \arcsin(-1/10)$$

$$\approx -.1$$

T' + | −

−.1

Maximum time when $\theta = .1$

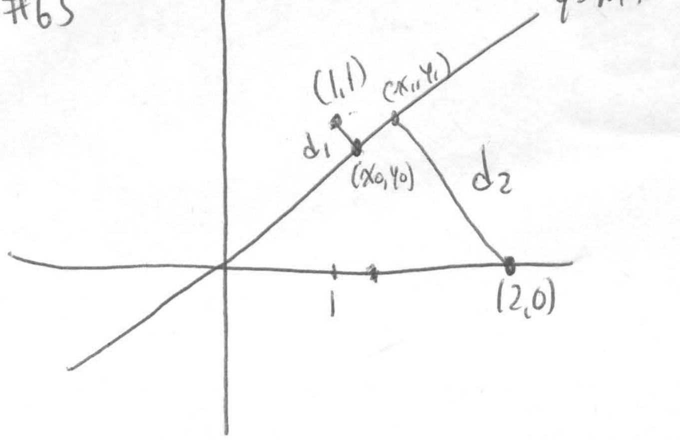
Min time when $\theta = \pi/2$

So, the lifeguard should just run the whole way.

Endpoints: $T(0) = \frac{2r}{v}$

$T(\pi/2) = \frac{\pi r}{10v}$ (Smallest)

#65



We want $d_1 = d_2$ to keep as far as possible away from both submarines

For this, we want to find m such that

$y_1 = m x_1$

$y_0 = m x_0$

and (x_0, y_0) is closest such point to $(1,1)$

and (x_1, y_1) is closest to $(2,0)$

Minimize d_1^2 (easier than minimizing d_1)

$$d_1^2 = (x_0 - 1)^2 + (m x_0 - 1)^2 = D_1$$

$$D_1' = 2(x_0 - 1) + 2m(m x_0 - 1) = x_0(2 + 2m^2) - 2(m+1) = 0 \text{ when } x_0 = \frac{m+1}{m^2+1}$$

For this x_0, we have
$$d_1^2 = \left(\frac{m+1-m^2-1}{m^2+1}\right)^2 + \left(\frac{m^2+m-m^2-1}{m^2+1}\right)^2 = \left(\frac{m(1-m)}{m^2+1}\right)^2 + \left(\frac{m-1}{m^2+1}\right)^2$$
$$= \frac{m^2(1-m)^2 + (m-1)^2}{(m^2+1)^2} = \frac{(m^2+1)(m-1)^2}{(m^2+1)^2} = \frac{(m-1)^2}{m^2+1}$$

$$d_2^2 = (x_1 - 2)^2 + y_1^2 = D_1 = (x_1 - 2)^2 + m^2 x_1^2$$

$$D_1' = 2(x_1 - 2) + 2m^2 x_1 = x_1(2 + 2m^2) - 4 = 0 \text{ when } x_1 = \frac{2}{m^2+1}$$

For this x_1 we have
$$d_2^2 = \left(\frac{2 - 2m^2 - 2}{m^2+1}\right)^2 + \left(\frac{2m}{m^2+1}\right)^2 = \frac{4m^4 + 4m^2}{(m^2+1)^2} = \frac{4m^2(m^2+1)}{(m^2+1)^2} = \frac{4m^2}{m^2+1}$$

Thus, $d_1^2 = d_2^2 \Rightarrow \frac{(m-1)^2}{m^2+1} = \frac{4m^2}{m^2+1} \Rightarrow m^2 - 2m + 1 = 4m^2 \Rightarrow 3m^2 + 2m - 1 = 0$

$\Rightarrow (3m - 1)(m + 1) = 0$

$\Rightarrow m = \frac{1}{3}$ or $m = -1$

But we need the line to go between the points, hence $m = \frac{1}{3}$.