Lab 11: What's Our Vector, Victor?

Goals: Improve communication, teamwork capacities and ability to record notes; Use graphical and analytical methods to add vectors to determine an unknown third force required to balance two known forces.

Equipment: Force Platform, 3 legs, 3 pulleys, central ring with string (note the care with which this is stored), 3 mass sets (keep track of each; don’t mix them up). Ruler and protractor. Note: in this lab, g is for grams, not the acceleration due to gravity.

Groups & Lab Notebook: For today’s investigation, you will work in groups of 3; your instructor will facilitate pair formation. Update your Table of Contents. General Lab Notes guidelines apply.

Lab Notebook Check Reminder
You will submit your Lab Notebook for a spot check on Monday November 10. Turn in your Lab Notebook by 5 pm Mon. Nov. 10 outside the door to Krishna’s office Lab 2 room 3255.

Part 1: Vector Addition of Forces

Getting Started:

a) Set up your Force Platform. For now, only use 2 pulleys.

b) Arrange 2 masses so that you have 90 g at 0° and 90 g at 180°. **Note: in this lab, g is for grams, not the acceleration due to gravity.** It should make sense that in this configuration, the ring will be in equilibrium: the tension in each string is equal (since the masses are equal) and they point in opposite directions (one tension pulls at 0° and the other tension pulls at 180°).

c) Now arrange 2 masses so that you have 90 g at 0° and 120 g at 180°. It should make sense that in this configuration, the ring cannot be in equilibrium. You will see that the ring is pressing against the center post; if you pull the ring away from the center post, it will accelerate in the direction of the unbalanced force (in this case towards 180°) and press against the center post again.

d) Now arrange the 2 masses so that you have 90 g at 0° and 120 g at 90°. This should be another unbalanced state. This time, with your hand, pull on the third string in a direction such that the ring is in equilibrium. You should see that you are pulling in a direction that points in the third quadrant. You need to pull in a direction that compensates for the pull along 0° and the pull along 90°.

Example: In the following extended example, we determine at what angle we need to put what mass in order to balance out 90 g at 0° and 120 g at 90° using a graphical approach and an analytical approach.

**Figure 1**

Figure 1 is a schematic representation of a bird’s eye view of the Force Platform; the lengths of the arrows are proportional to the magnitude of the force (not the lengths of the string) and drawn to scale. We can see the center ring represented. Figure 1 shows a scale diagram of vector \( \vec{A} \) (which has magnitude “90 g”) and vector \( \vec{B} \) (which has magnitude “120 g”). The scaling factor is 1 cm = 60 g, thus 1.5 cm = 90 g and 2.0 cm = 120 g. We want to find the vector \( \vec{C} \) such that \( \vec{A} + \vec{B} + \vec{C} = 0 \). (Note that grams are a unit of mass and are not strictly speaking a unit of force, but the actual unit of force here is proportional to the mass.)

In Figure 2, we have rearranged vectors \( \vec{A} \) and \( \vec{B} \). As long as we don’t change the length and direction of the vectors, this is a mathematically valid move. We see that \( \vec{A} + \vec{B} \) does not equal 0, and can picture what the vector \( \vec{C} \) would need to be in order for the sum of the 3 vectors to equal 0.

**Figure 2**

**Figure 3**

Figure 3 shows \( \vec{C} \), and shows how \( \vec{A} + \vec{B} + \vec{C} = 0 \), as required. From this figure, we could measure the length of \( \vec{C} \) and use the scaling factor 1 cm = 60 g to figure out the mass required. We can also measure the angle to find the direction of \( \vec{C} \).
In your lab notebook (or on a separate sheet of paper which you will attach to your lab notebook), reproduce your own version of Figure 3. Use a more reasonable scaling factor that takes advantage of the larger space you have available. For example, try a scaling factor of 1 cm = 10 g. Use your ruler and protractor to determine the length (measured in cm but converted to g using the scaling factor) and direction (angle in degrees) of \( \vec{C} \).

Figure 4 returns to the schematic arrangement as in Figure 1, and shows each of \( \vec{A}, \vec{B}, \) and \( \vec{C} \) as they would be arranged on the force table when viewed from above. From your own diagram, you should know how much mass to use and in what direction (what angle) to place it so that the ring will be in equilibrium when you hang 90 g at 0\(^\circ\) and 120 g at 90\(^\circ\).

Test this out. You’ll know if it worked!

Vector diagrams are powerful and useful tools. Even more powerful is the analytical approach, which uses trigonometry.

The tables below summarize the results of the analytical approach. As needed, ask an instructor to go over this approach with you. Recall that the goal is to have \( \vec{A} + \vec{B} + \vec{C} = 0 \) because we are looking for the equilibrium condition.

We begin by calculating the \( x \) and \( y \) components of the given vectors:

<table>
<thead>
<tr>
<th>given information</th>
<th>( \vec{A} )</th>
<th>( \vec{B} )</th>
<th>( \vec{C} )</th>
<th>( \vec{R} = \vec{A} + \vec{B} + \vec{C} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>magnitude (g)</td>
<td>( +90 )</td>
<td>( +120 )</td>
<td>( C_x )</td>
<td>( R_x = A_x + B_x + C_x = 0 )</td>
</tr>
<tr>
<td>direction (deg)</td>
<td>0</td>
<td>90</td>
<td>( C_y )</td>
<td>( R_y = A_y + B_y + C_y = 0 )</td>
</tr>
<tr>
<td>( A_x )</td>
<td>( +90 )</td>
<td>( 0 )</td>
<td>( C_x )</td>
<td>( R_x = A_x + B_x + C_x = 0 )</td>
</tr>
<tr>
<td>( A_y )</td>
<td>( +120 )</td>
<td>( +120 )</td>
<td>( C_y )</td>
<td>( R_y = A_y + B_y + C_y = 0 )</td>
</tr>
</tbody>
</table>

In order for \( \vec{R} = \vec{A} + \vec{B} + \vec{C} = 0 \), then each component of \( \vec{R} \) must = 0 as well. From this, can see that \( C_x = -90 \text{ g} \) and \( C_y = -120 \text{ g} \).

We organize by components, and introduce the resultant vector \( \vec{R} = \vec{A} + \vec{B} + \vec{C} \) (recall that \( \vec{A} + \vec{B} + \vec{C} = 0 \), so \( \vec{R} = 0 \) since we are looking for the equilibrium condition):

Now that we have the components of \( \vec{C} \), we can sketch the vector.

Use Pythagorean Theorem to get magnitude of \( \vec{C} \):

\[ |\vec{C}| = C = \sqrt{(-90 \text{ g})^2 + (-120 \text{ g})^2} = 150 \text{ g} \]

Use trigonometry to get direction of \( \vec{C} \):

\[ \theta = \tan^{-1} \left( \frac{C_y}{C_x} \right) \pm 180^\circ = \tan^{-1} \left( \frac{-120 \text{ g}}{-90 \text{ g}} \right) \pm 180^\circ = 53.13^\circ \pm 180^\circ \text{?} \]

Recall that inverse tangent on a standard calculator can’t differentiate between an angle in the first quadrant vs. the third quadrant or an angle in the second quadrant vs. the fourth. So you have to use a figure or reasoning to determine whether to add (or subtract) 180\(^\circ\) or \( \pi \) radians from the result of your calculator.

From the figure or the components, can see that the vector points in the third quadrant, so yes +180\(^\circ\) \( \rightarrow \) 53\(^\circ\) + 180\(^\circ\) = 233\(^\circ\).
Try it yourself: For each of the 3 cases below:
- draw a vector diagram (to scale and as careful as possible) to determine the angle and amount for the third mass to balance out the given 2; and
- use the analytic approach to determine the angle and amount for the third mass in order to balance out the given 2; and
- test your result for each case – you’ll know if you did it correctly!

Case 1: 80 g at 0°, 60 g at 270°.

Case 2: 50 g at 30°, 100 g at 135°.

Case 3: 100 g at 15°, 100 g at 135°.

Analysis:
a) Briefly discuss why we can use the mass on the hangars for the tension in the strings.
b) You might have seen a symmetry that allowed you to take a short-cut in Case 3. If you took advantage of the symmetry – great!, but now go back and practice the full analytical method. If you didn't see the symmetry, look at your vector diagram as well as the results of your analytical method and see if you can figure it out.
c) In Case 2, you had to resolve 2 vectors into components. What if instead you had rotated your vector diagram by 30° in the clockwise direction? Then one of the vectors would have been easy to resolve into components. Try this approach and see what happens. Note that this means the 100 g mass is now at 105° in the new coordinate system.
d) Look at the Case 2 numbers again. One way to think about this is that you use 50 g to balance 100 g + 100 g. In effect, you are using 50 g to balance 200 g. How is this possible?

Part 2: Two Dimensional Motion
In the program share, under Handouts: Lab 11, you will find some LoggerPro files.
a) Begin by opening Basketball Shot with Analysis. In the Replay window, press Start. You can adjust the Speed in the Replay window if you’d like. Watch several times, making sure you can connect the motion of the ball shown in the movie with the corresponding x position vs. time and y position vs. time graphs. Write some brief notes. Close this file; there’s nothing to save.
b) Next, open Basketball Shot with Vector Analysis. In the Replay window, press Start, adjusting the Speed if you’d like. In this version, the velocity vector for the basketball is shown at various points along its trajectory, along with its x and y components. What do you notice about the x component of the velocity? What do you notice about the y component of the velocity? What connections can you make to what you’ve read about projectile motion? Write some brief notes. Close this file; there’s nothing to save.

Part 3: Fan Cart with Friction
(Not directly related to this lab, but a required analysis to follow up on Lab 9). In Lab 9, you obtained data for the fan cart with friction pad. In this data, you should have seen a clear asymmetry in the two “parts” of the trip: moving towards the motion detector vs. moving away from the motion detector (or before the turn-around point vs. after the turn-around point). We attributed the difference in acceleration to the effects of friction. Determine the magnitude of the fan force (assumed constant) and the magnitude of the friction force (assumed constant in magnitude though not direction).

Part 4: Half-Atwood Machine
(Not directly related to this lab, but a required analysis to follow up on Lab 9). In Lab 9, you obtained data for a half-Atwood machine for varying combinations of hanging mass and cart mass. Subsequently, we analyzed the half-Atwood machine in great detail, determining a formula for the acceleration as a function of the two masses. Make sure you can do that analysis on your own, starting from free body diagrams. Compare the experimental results to the formula prediction, and discuss consistence and deviations.