Lab 1: Graphing Functions

Goals: Start learning to use the online graphing calculus Desmos; review/explore 3 important functions: linear, quadratic, and sine;

Part –1: Program Website
If you haven’t already, spend a few minutes exploring the program website sites.evergreen.edu/motion; make sure to look at older entries.

Part 0. Lab Notebook (LNb)
- If you don’t have your LNb yet, take notes on separate paper which you can later insert into (or copy from) your LNb.
- Your name and contact information should be written prominently and early.
- You should leave room for a Table of Contents. If you have already begun to write on the first page, then you can insert a separate sheet of paper for a Table of Contents; ask how if you are uncertain.
- Each new lab should begin on a new page, and start with the title of the investigation. You should also include the names and contact information of any lab partners.
- It’s a good idea to leave some room at the end of each lab entry in case you need to add something later.
- For this lab, you should leave sufficient space to include any graphs which you may choose to print out and tape into your LNb.
- Take notes which are useful to you on your observations.

Part 1: Introduction to Desmos
1. Launch a web browser and go to desmos.com.
2. On the main desmos.com page you can get the quick-start guide (a quick and handy pdf). Do this before you launch the desmos calculator.
3. Launch Desmos and take a quick scan of the layout. Match up the parts of the screen with the quick-start guide.
4. Click on the question mark (?) in the upper right of the Desmos calculator. You can get three kinds of guides for Desmos: Tours (there are 4 of them that you can click on), a Desmos User Guide under Resources, and Knowledge Base under Resources. You’ll use each of these later in the lab. Open up the User Guide so you have it available for the rest of the lab. It should open as a separate window or tab.
5. Now open up the Knowledge base. Search for video tutorials in the knowledge base. Among the videos you should see a How To: Sliders video tutorial and a How to: Moveable Points video tutorial. You’ll use these later.
6. You can easily create a Desmos account that allows you to save work and share work. You don’t need to do this, but it is available to you if later you want to save your work. To create an account, look for the create account button. As far as we have noticed, creating a Desmos account does not appear to be very intrusive.

Part 2: Graphing Functions
1. Here, we will explore the effects that changing the parameters $m$ and $b$ have on the graph of the straight line $f(x) = mx + b$.
   a) The plot provided is for $f(x) = 1x + 0 = x$, so that $m = 1$ and $b = 0$. Use Desmos to reproduce a version of this graph, including the scale of the axes and the spacing of the gridlines (you might not be able to reproduce the placement of the numbers on the axes or their exact values, that is fine).
   b) How does changing just the value of $b$ affect your graph? Plot $f(x) = x + b$ for different values of $b$. Try at least the following values for $b$: 0, ±0.25, ±0.5, ±1, ±2, ±4, ±8. Make one single graph that shows all these lines (you will have multiple equations entered). Describe the effects that varying $b$ had on the line. What aspect of the line is determined by the parameter $b$?
   c) How does changing just the value of $m$ affect your graph? Plot $y = f(x) = mx$ for different values of $m$. Try at least the following values for $m$: 0, ±0.25, ±0.5, ±1, ±2, ±4, ±8. Make one single graph that shows all these lines (you will have multiple equations entered). Describe the effects that varying $m$ had on the graph. What aspect of the line is determined by the parameter $m$?
   d) Review the How To: Sliders video tutorial and make $f(x) = mx + b$ with $m$ and $b$ as sliders. Play with the sliders and see how what you observe matches your work in parts c) and d)

1This exercise is based on Lab 1: Graphing Functions, in Learning by Discovery: A Lab Manual for Calculus, ed. Anita Solow, 1993, The Mathematical Association of America.
2. Next, we will explore the effects that changing the parameters \( A \), \( B \), and \( C \) have on the graph of the parabola \( f(x) = Ax^2 + Bx + C \).

a) The plot provided is for \( f(x) = 1x^2 + 0x + 0 = x^2 \), so that \( A = 1 \), \( B = 0 \), and \( C = 0 \). Use Desmos to reproduce a version of this graph, including the scale of the axes and the lack of grid-lines.

b) Make another \( f(x) = Ax^2 + Bx + C \) with sliders \( A \), \( B \), and \( C \). How does changing just the value of \( C \) affect your graph? Describe the effects that varying \( C \) had on the parabola. What part of the parabola is determined by \( C \)?

c) How does changing just the value of \( B \) affect your graph? Describe the effects that varying \( B \) had on the parabola. What part of the parabola is determined by \( B \)?

d) How does changing just the value of \( A \) affect your graph? Describe the effects that varying \( A \) had on the parabola.

e) Summarize your results. How does changing the parameters \( A \), \( B \), and \( C \) one at a time affect the shape of the parabola? The vertex? The y-intercept?

f) How can we explain these observation? One way might be to consider \( Ax^2 + Bx + C \) as the sum of \( 2Ax \) (whose behavior we can understand pretty easily) and \( Bx + C \), which is just the equation for a straight line. Does this insight help you to understand the observed behavior? Another approach might be to consider changing the equation to vertex form (if you look anything up, note your sources). Does changing to vertex form help you understand the observed behavior? Yet one more way might be to consider using the quadratic equation.

3. Now, we will explore the effects that changing the parameters \( A \), \( B \), and \( C \) have on the graph of the sine function \( f(x) = A\sin(Bx + C) \).

a) The plot provided is for \( f(x) = \sin(1x + 0) = \sin(x) \), so that \( A = 1 \), \( B = 1 \), and \( C = 0 \), for \(-2\pi \leq x \leq 2\pi \). Use Desmos to reproduce this graph, including the scale of the axes.

b) As before, test out the effects of independently varying \( A \), \( B \), and \( C \). Summarize your results. How does affecting the parameters \( A \), \( B \), and \( C \) one at a time affect the shape of the sine curve? Which parameters control the amplitude (height)? The period (repeat time)? Any horizontal shifting?

FURTHER EXPLORATIONS (if time)

4. How can we explain the observations about the effects of the parameters \( A \), \( B \), and \( C \) on the graph of the parabola \( f(x) = Ax^2 + Bx + C \) that you observed in 2. above? One way might be to consider \( Ax^2 + Bx + C \) as the sum of \( 2Ax \) (whose behavior we can understand pretty easily) and \( Bx + C \), which is just the equation for a straight line. Does this insight help you to understand the observed behavior? Another approach might be to consider changing the equation to vertex form (if you look anything up, note your sources). Does changing to vertex form help you understand the observed behavior? Yet one more way might be to consider using the quadratic equation.

5. Consider the quadratic polynomial \( kx^2 + (k + 1)x - (k + 2) \), where \( k \) is a constant.

a) Graph the parabolas \( y = x^2 + 2x - 3 \) (where \( k = 1 \)) and \( y = 2x^2 + 3x - 4 \) (where \( k = 2 \)). Also graph the parabolas for \( k = 3, 4 \), and a few larger values. Compare the vertices and general shapes of these parabolas. What is happening as \( k \) increases?

b) Notice that all of these parabolas cross the x-axis at two points. Let \( s(k) \) be the larger of the two solutions of \( kx^2 + (k + 1)x - (k + 2) = 0 \) (in other words, the larger of the two x-intercepts). By looking at the graphs (zooming in as needed), approximate \( s(1), s(2), s(3), \) and \( s(4) \).

c) Guess the behavior of \( s(k) \) as \( k \) gets very large. Check your guess graphically.

d) Use the quadratic formula to find the roots of the polynomial \( kx^2 + (k + 1)x - (k + 2) \). Use this formula to verify your answer to part c).