1. A satellite in a Circular Orbit

A satellite, mass m_1, orbits a planet of mass m_2 at distance r from the planet's center. $m_1 << m_2$

A. The info given suffices to give unique values for V, U_g, L of satellite

$\boxed{\text{TRUE}}$ $U_g = \dfrac{-Gm_1m_2}{r}$, $L = m_1vr$, $V = \sqrt{\dfrac{GM_2}{r}}$

B. The total mechanical energy is conserved

$\boxed{\text{TRUE}}$ No dissipative forces act on the system

C. The linear momentum vector of the satellite is conserved.

$\boxed{\text{FALSE}}$ Since $\vec{v}$ changes direction, $\vec{p} = m\vec{v}$ is always changing as well. This is because there is a nonzero net force on the satellite, & N2L says $\vec{F}_{net} = \dfrac{d\vec{p}}{dt} \neq 0$

D. $\vec{L}$ of satellite about planet's center is conserved

$\boxed{\text{TRUE}}$ Because $\vec{F}_g$ is directed toward the center, its torque is zero ($\vec{\tau} = \vec{r} \times \vec{F}$, but $\vec{r}$ & $\vec{F}$ are $180°$ apart, so $|\vec{r} \times \vec{F}| = rF\sin\theta = rF\sin 180° = 0$)

Since $\vec{\tau}_{net} = \dfrac{d\vec{L}}{dt}$, $\vec{L}$ is constant $\leftrightarrow$ $\vec{L}$ is conserved

E. One can calculate orbital speed from conservation of E & $\vec{p}$

$\boxed{\text{FALSE}}$ For one thing, $\vec{p}$ is <u>not</u> conserved (see C.)

2. Ex 8.30 $V_{esc} = 7.5\ \frac{km}{s}$ for a planet whose mass is 3.3×10^{24} kg. What is the planet's radius?

Since $V_{esc} = \sqrt{\dfrac{2GM}{r}}$, $V_{esc}^2 = \dfrac{2GM}{r} \rightarrow r = \dfrac{2GM}{V_{esc}^2} = \dfrac{2(6.67\times10^{-11}\,N\frac{m^2}{kg^2})(3.3\times10^{24}\,kg)}{(7.5\times10^3\,\frac{m}{s})^2}$

$r = 7.83 \times 10^6\,m \rightarrow \boxed{7.8 \times 10^6\,m}$

3. Two identical spheres, $m = 7.75\,kg$, experience $F_g = 0.285\,\mu N$. Alt Ex 8.77
What is the separation between their centers?

$$F_G = \frac{G m_1 m_2}{r^2} \rightarrow r^2 = \frac{G m_1 m_2}{F_G} \rightarrow r = \sqrt{\frac{G m_1 m_2}{F_G}} = \sqrt{\frac{6.67\times10^{-11}\,N\frac{m^2}{kg^2}(7.75\,kg)^2}{0.285\times10^{-6}\,N}}$$

$\boxed{r = 0.119\,m}$ or 11.9 cm

4. CQ 8.01 A baseball is at Earth's surface. True statements include:

The grav. force of the ball on Earth is exactly the same as the force by Earth on ball (in magnitude). True by N3L

5. CQ 8.04
Suppose Earth (& everything on it) suddely doubled in density (mass doubles, size does not change). How would g change?

Since $g = \frac{GM}{r^2}$, doubling M (only) $\boxed{\text{doubles } g}$

6. Orbital Speed of a Satellite

A. Satellite 1: speed $= v$, orbit radius r
satellite 2: speed $< v$ How does its orbital radius compare to r?

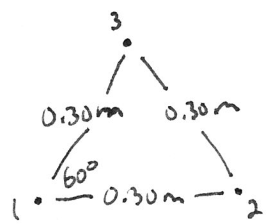
Since $v = \sqrt{\frac{GM}{r}}$, smaller v implies larger r → $\boxed{\text{Distance} > r}$

B. Satellite 1 orbits Earth at r, v_E
" " Moon at r, v_m
} compare T_E to T_m

Again, since $v = \sqrt{\frac{GM}{r}}$ and $M_E > M_m$, the satellite in Earth orbit has greater speed & thus shorter period. So $T_E < T_m$ → $\boxed{T_m > T_E}$

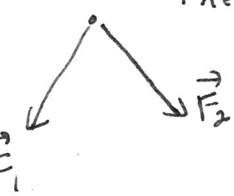
7. Prob. 8.03

3 identical 50 kg masses are at vertices of equilateral triangle. One is released from rest. What is its initial acceleration?



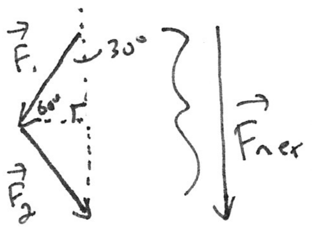
Let's release #3. Apply N2L

FBD:

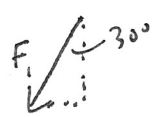


$\vec{F}_{net} = m\vec{a}$

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$ (by definition)



The x components of $\vec{F}_1$ & $\vec{F}_2$ are going to cancel by symmetry. Meanwhile the magnitude of $F_{net} = |F_{1y}| + |F_{2y}|$. Also note that $F_1 = F_2 = F_g$ – both individual vectors will have the same magnitude, since all masses and distances are equal. Let's get the y component of one of them



$F_{1y} = F_1 \cos 30° = \dfrac{G m_1 m_3}{r^2} \cos 30° = \dfrac{G m^2}{r^2} \cos 30°$ (since $m_1 = m_3 = m = m_2$)

$\vec{F}_{net} = |F_{1y}| + |F_{2y}| = 2|F_{1y}| = 2 \dfrac{G m^2}{r^2} \cos 30°$ $\cancel{= 2166.62\times}$

Since $F_{net} = ma$, $a = \dfrac{F_{net}}{m} = \dfrac{1}{m} 2 \dfrac{Gm^2}{r^2} \cos 30° = \dfrac{2GM}{r^2} \cos 30°$

$$a = \frac{2\left(6.67\times10^{-11} N \frac{m^2}{kg^2}\right)(50 kg)}{(0.3 m)^2} \frac{\sqrt{3}}{2} = \boxed{6.4\times10^{-8} \frac{m}{s^2}}$$

8. Problem 8.07 By how many Newtons does the weight of a $m = 100\,kg$ person change going from sea level to 5.0 km?

$$F_G = \frac{G M_E m}{r^2} \qquad \Delta F_G = GMm\left(\frac{1}{(r_E + 5\,km)^2} - \frac{1}{r_E^2}\right)$$

$$\Delta F_G = \left(6.67\times10^{-11}\,N\frac{m^2}{kg^2}\right)\left(5.97\times10^{24}\,kg\right)(100\,kg)\left(\frac{1}{\left[(6.37\times10^6 + 5\times10^3)\,m\right]^2} - \frac{1}{(6.37\times10^6\,m)^2}\right)$$

$$\boxed{\Delta F_g = -1.5\,N}$$

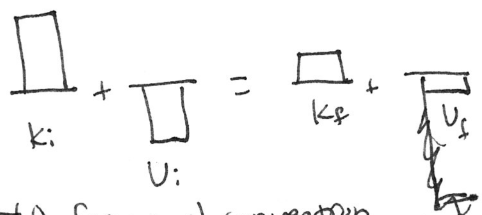
Change should be negative, and a small fraction of weight (since 5km is a small fraction of 6400 km)

9. Problem 8.15

Planet w/ $M = 2.92\times10^{23}\,kg$, $R = 5.00\times10^6\,m$. Projectile fired straight up at 2000 m/s.

What is speed when $h = 1000\,km$?

Apply conservation of mechanical energy



$K = \frac{1}{2}mv^2$, $U_g = \frac{-GMm}{r}$

Note that $U_i \neq 0$ for usual convention

$$\underbrace{\frac{1}{2}mv_i^2}_{K_i} - \underbrace{\frac{GMm}{R}}_{U_i} = \underbrace{\frac{1}{2}mv_f^2}_{K_f} - \underbrace{\frac{GMm}{(R + 1000\,km)}}_{U_f}$$

Multiply through by $\frac{2}{m}$...

$$v_f^2 = v_i^2 + 2GM\left(\frac{1}{R+h} - \frac{1}{R}\right)$$

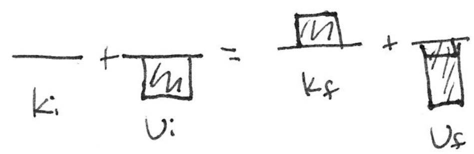
$$v_f = \sqrt{v_i^2 + 2GM\left(\frac{1}{R+h} - \frac{1}{R}\right)} = \sqrt{\left(2000\frac{m}{s}\right)^2 + 2\left(6.67\times10^{-11}\,N\frac{m^2}{kg^2}\right)\left(2.92\times10^{23}\,kg\right)\left[\frac{1}{6\times10^6\,m} - \frac{1}{5\times10^6\,m}\right]}$$

$$= 1.64\times10^3\,\frac{m}{s} \qquad \boxed{v_f = 1640\,\frac{m}{s}}$$

10. Problem 8.18

A 910 kg object is released from h = 1200 km. What is speed when it strikes Earth (ignoring air resistance)?

Just like # 9... $K_i = 0$



$$0 - \frac{GM\cancel{m}}{(R_E + h)} = \frac{1}{2}\cancel{m}v^2 - \frac{GM\cancel{m}}{R_E}$$

$$v^2 = 2GM\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right)$$

$$\downarrow$$

$$v = \sqrt{2GM\left(\frac{1}{R_E} - \frac{1}{R_E + h}\right)}$$

$$= \sqrt{2\left(6.67 \times 10^{-11} \frac{N m^2}{kg^2}\right)\left(5.97 \times 10^{24} kg\right)\left[\frac{1}{6.37 \times 10^6 m} - \frac{1}{(6.37 \times 10^6 m + 1.2 \times 10^6 m)}\right]}$$

$$= \sqrt{1.98 \times 10^7 \frac{m^2}{s^2}} = 4.45 \times 10^3 \frac{m}{s} \rightarrow \boxed{4.5 \frac{km}{s}}$$

11. Problem 8.34

A high jumper can jump 2.45 m on Earth. How high would he jump on

A. Mars ($g_M = 3.74 \frac{m}{s^2}$)

We'll assume he does the same work in jumping, which will convert to gravitational potential energy change. Then

$$\cancel{m} g_E h_E = \cancel{m} g_{mars} h_{mars}$$

$$h_{mars} = h_E \frac{g_E}{g_{mars}} = 2.45\,m \left(\frac{9.81\,m/s^2}{3.74\,m/s^2} \right) = \boxed{6.43\,m}$$

B. Moon ($g_m = 1.62\,m/s^2$)

As above, only ~~us~~ using g_{moon}

$$h_{moon} = 2.45\,m \left(\frac{9.81\,m/s^2}{1.62\,m/s^2} \right) = 1.48 \times 10^1\,m = \boxed{14.8\,m}$$