Week 11 HW Solutions (Physics)

(1.) Ex 13.19

A. Find $x(t)$ for 12.5cm amplitude, $f = 6.68\,Hz$, $x(t=0) = A$

General formula is $x(t) = A\cos(\omega t + \phi)$

$$x(0) = A = A\cos(\omega \overset{t=0}{t} + \phi) \rightarrow \cos\phi = 1 \rightarrow \phi = 0$$

$$f = 6.68\,Hz \rightarrow \omega = 2\pi f = 2\pi(6.68\,Hz) = 42.0\,s^{-1}$$

$$\therefore \boxed{x(t) = (12.5\,cm)\cos(42.0\,s^{-1}\,t)}$$

B. Find $x(t)$ for amplitude 2.15cm, $\omega = 4.63\,s^{-1}$, $|V_{max}|$ at $t=0$

Since $x(t) = A\cos(\omega t + \phi)$

$$V = \frac{dx}{dt} = -(A\omega)\sin(\omega t + \phi)$$

Since $|V_{max}|$ is at $t=0$, we know $\phi = \pm\frac{\pi}{2}$. So

$$x(t) = A\cos\left(\omega t \pm \frac{\pi}{2}\right)$$

But $\cos\left(\theta \pm \frac{\pi}{2}\right)$ can also be written as $\pm\sin\theta$. So

$$x(t) = \pm A\sin\omega t = \boxed{\pm(2.15\,cm)\sin\left[(4.63\,s^{-1})t\right]}$$

(2.) Ex. 13.20 A skyscraper oscillates 31 times in 10 minutes

A. What is T?

$$T = \frac{\text{time}}{\#\text{ oscillations}} = \frac{(10\,min)\left(\frac{60\,s}{1\,min}\right)}{31\text{ oscillations}} = \boxed{19\,s}$$

B. Find f in Hz

$$f = \frac{1}{T} = \frac{1}{19\,s} = 0.052\,Hz$$

(3) Ex 13.21 A hummingbird's wings vibrate at 42Hz. What is T?

$$T = \frac{1}{f} = \frac{1}{42\,Hz} = \frac{1}{42s^{-1}} = \boxed{0.024s}$$

(4) Ex 13.27
A particle undergoes SHM with $V_{max} = 1.2\,m/s$, $a_{max} = 3.3\,m/s^2$

A. Find ω Since $x(t) = A\cos(\omega t + \phi)$

$$v(t) = \frac{dx}{dt} = -(A\omega)\sin(\omega t + \phi) = -V_{max}\sin(\omega t + \phi)$$

$$a(t) = \frac{d^2x}{dt^2} = \frac{dv}{dt} = -A\omega^2\cos(\omega t + \phi) = -a_{max}\cos(\omega t + \phi)$$

So $\left.\begin{array}{l} V_{max} = A\omega \\ a_{max} = A\omega^2 \end{array}\right\} \rightarrow \frac{a_{max}}{V_{max}} = \frac{\cancel{A}\omega^2}{\cancel{A}\omega} = \omega = \frac{3.3\,m/s^2}{1.2\,m/s} = \boxed{2.8\,s^{-1}}$

B. Find the period $T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{2.8s^{-1}} = \boxed{2.3s}$

C. Since $V_{max} = A\omega$, $A = \frac{V_{max}}{\omega} = \frac{1.2\,m/s}{2.8s^{-1}} = \boxed{0.44m}$

(5) Ex 13.32 A wheel rotates at $800\,\frac{rev}{min}$

A. What is f in Hz? $f = \frac{800\,rev}{min} \cdot \frac{1\,min}{60s} = 13.3\,\frac{rev}{s} = \boxed{13.3\,Hz}$

B. What is ω? $\omega = 2\pi f = 2\pi(13.3\,Hz) = \boxed{83.8\,s^{-1}}$

(6) Ex 13.34 A 600g mass on a spring oscillates at 1.2 Hz. Total energy is 0.51J.

What is the amplitude?

Since we don't know the spring constant, we can't use U_{sp} to get an energy formula. So let's focus on the moment when the spring is unstretched & $V = V_{max}$.

Since $V(t) = -(A\omega)\sin(\omega t + \phi) = -V_{max}\sin(\omega t + \phi)$, we know $V_{max} = A\omega$.

Plug this ~~value~~ expression into the kinetic energy formula:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(A\omega)^2 \rightarrow m(A\omega)^2 = 2K$$

$$(A\omega)^2 = \frac{2K}{m}$$

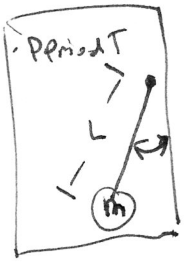
$$A\omega = \sqrt{\frac{2K}{m}}$$

$$A = \frac{1}{\omega}\sqrt{\frac{2K}{m}} = \frac{1}{2\pi f}\sqrt{\frac{2K}{m}}$$

$$= \frac{1}{2\pi(1.2Hz)}\sqrt{\frac{2(0.51J)}{0.6\,kg}}$$

$$\boxed{A = 0.17m}$$

7 Changing Period of a Pendulum



A. If the bob's mass doubles, what is the new period?

$$T = 2\pi\sqrt{\frac{L}{g}} \rightarrow \text{mass doesn't matter. So } T \rightarrow \boxed{T}$$

B. How about on the moon, where $g = \frac{1}{6} g_{Earth}$?

$$\frac{T_{moon}}{T_{Earth}} = \frac{\cancel{2\pi}\sqrt{\frac{\cancel{L}}{g_{moon}}}}{\cancel{2\pi}\sqrt{\frac{\cancel{L}}{g_{Earth}}}} = \frac{\sqrt{\frac{1}{g_{moon}}}}{\sqrt{\frac{1}{g_{Earth}}}} = \sqrt{\frac{g_E}{g_m}} = \sqrt{\frac{g_E}{\left(\frac{g_E}{6}\right)}} = \sqrt{6}$$

$$\therefore\ T_{moon} = \sqrt{6}\, T_{Earth} = \sqrt{6}\, T$$

C. What about taking it to an orbiting space station?

No oscillation because bob & attachment point are both in freefall

Note that in low Earth orbit, there is still substantial gravitational force (that is what holds satellites in orbit!) You'll see how to calculate this in Chapter 8.

8 CQ 13.01 Which describe SHM? General form: $A\cos(\omega t + \phi)$

SHM:
- $x = 2\cos(3t - 1)$
- $x = 8\cos 3t$
- $x = 5\sin 3t$ ← since $\sin\phi = \cos\left(\phi \pm \frac{\pi}{2}\right)$

Not SHM:
- $x = 4\tan 2t$
- $x = 5\sin^2 3t$

Graph these to see why not!

9 CQ 13.03 For an object in SHM...

✓ $a = 0$ when $v = v_{max}$ since $v(t) = -A\omega\sin(\omega t + \phi)$ & $a(t) = -A\omega^2\cos(\omega t + \phi)$ when $\sin(\omega t + \phi) = 1$, $\cos(\omega t + \phi) = 0$

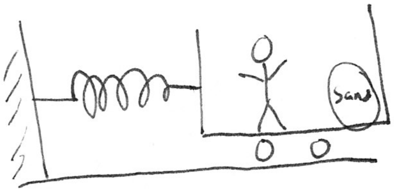
✓ $a = a_{max}$ when $x = x_{max}$ since $a(t) = -A\omega^2\cos(\omega t + \phi)$ & $x(t) = A\cos(\omega t + \phi)$ both peak when $\cos(\omega t + \phi) = 1$

✓ $a = a_{max}$ when $v = 0$ because when $\sin(\omega t + \phi) = 0$, $\cos(\omega t + \phi) = 1$

X $a = a_{max}$ when $x = 0$ is false (see

X $a = a_{max}$ when $v = v_{max}$ is false (see

(10) Mass & SHM CQ Shaker cart



$x(t) = A\cos(\omega t + \phi)$

A. How does dropping sand bag at $X_{equilibrium}$ affect A?

This is like comparing SHM with same V_{max} but decreasing mass. This means the system has less kinetic energy than it did with the ~~[illegible]~~ sandbag present. Given the same spring, and less energy, the amplitude will be smaller because less energy is available to stretch the spring

B. How does dropping the sandbag at $X_{equilibrium}$ affect V_{max}?

As long as the bag is simply dropped & not thrown, the speed of the cart does not change. Since V_{max} occurs at $X_{equilibrium}$, the old V_{max} is the new V_{max} – there is no change

C. How does dropping the sand bag at $X = A$ affect A?

Again, assuming a "clean" drop, the motion is no different from what it would be releasing the cart from rest at that location – which would give the same amplitude. So there is no change in amplitude.

D. How does dropping the bag at $X = A$ ~~[illegible]~~ affect V_{max}?

Because, at $X = A$, $V = 0$, all the energy of the system is in U_{sp}.

~~But at X_{equil}~~, At $X_{equilibrium}$, we therefore expect the same energy.

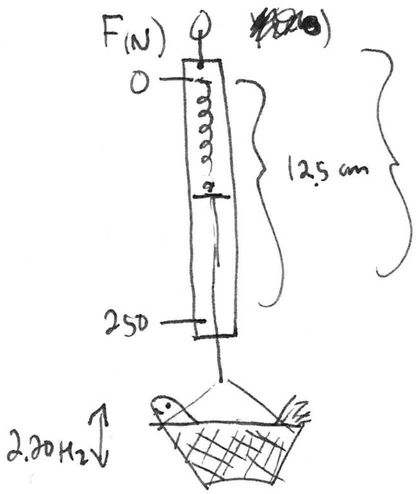
But there is now less mass, so the only way to have the same E is for v to be larger:

$$\frac{1}{2}(\underset{\substack{\uparrow \\ \text{cart}}}{m_c} + \underset{\substack{\uparrow \\ \text{bag}}}{m_b})V_{c+b}^2 = \frac{1}{2}m_c V_c^2 \rightarrow V_c^2 = \frac{m_c + m_b}{m_c}V_{c+b}^2$$

$$V_c = \sqrt{\frac{m_c + m_b}{m_c}}\,V_{c+b} > V_{c+b}$$

So V_{max} increases

(11.) The Fish Scale



A scale reads 0 → 250 N and has a readout 12.5 cm long.

A fish oscillates at 2.20 Hz.

implies spring constant

What is the mass of the fish?

First, we need k: Use Hooke's Law $F_{sp} = -kx$

$$k = \frac{\Delta F}{\Delta x} = \frac{250\,N}{0.125\,m} = 2000\,\frac{N}{m}$$

Next, note that for a mass-spring system

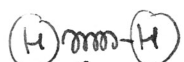
$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \qquad k, f \text{ known, } m \text{ unknown} \rightarrow \text{can solve}$$

$$\frac{k}{m} = 4\pi^2 f^2$$

$$m = \frac{k}{4\pi^2 f^2} = \frac{2000\,\frac{N}{m}}{4\pi^2 (2.20\,Hz)^2} = 10.5\,\frac{N}{m\,Hz^2}$$

Check units $\frac{N}{m\,Hz^2} = \frac{kg\,\cancel{\frac{m}{s^2}}}{\cancel{m}\,\cancel{\frac{1}{s^2}}} = kg$ ✓ $\boxed{m = 10.5\,kg}$

(12.) H_2 Vibration



$k = 520\,\frac{N}{m}$

$m = 1.661 \times 10^{-27}\,kg$

Note that for a system of 2 equal masses floating in space, connected by a spring and oscillating, the "effective" mass one plugs into the formula for ω is $\frac{m}{2}$

$$\omega = \sqrt{\frac{k}{m_{eff}}} = \sqrt{\frac{k}{\frac{1}{2}m}} = \sqrt{\frac{2k}{m}}$$

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{2k}{m}}} = 2\pi\sqrt{\frac{m}{2k}} = 2\pi\sqrt{\frac{1.661 \times 10^{-27}\,kg}{2(520\,\frac{N}{m})}} = 7.94 \times 10^{-15}\sqrt{\frac{kg}{\left(\frac{N}{m}\right)}}$$

Check units! $\sqrt{\frac{kg}{\left(\frac{N}{m}\right)}} = \sqrt{\frac{kg\,m}{N}} = \sqrt{\frac{\cancel{kg}\,\cancel{m}}{\cancel{kg}\,\frac{\cancel{m}}{s^2}}}$

$$= \sqrt{\frac{1}{\left(\frac{1}{s^2}\right)}} = \sqrt{s^2} = s \checkmark$$

$\therefore T = 7.94 \times 10^{-15}\,s$

$\boxed{T = 7.94\,fs}$

Wait... the question asked for period in Hz! No worries...

$$T = \frac{1}{f} = \frac{1}{7.94 \times 10^{-15}\,s} = \boxed{1.25 \times 10^{14}\ Hz} = 125\ THz$$

(13.) Alt Ex 13.92 An HCl molecule completes one oscillation in 11.6 fs. What is f?

$$T = 11.6 \times 10^{-15}\,s \qquad f = \frac{1}{T} = \frac{1}{11.6 \times 10^{-15}\,s} = 8.62 \times 10^{13}\ Hz = \boxed{86.2\ THz}$$

These two problems show that many molecular vibrations have frequencies best measured in terahertz (THz) & periods in femtoseconds (fs)

(14.) Alt Ex 13.94
Quartz crystal, $f = 32{,}768\ Hz$; $a_{max} = 6.22\ \frac{km}{s^2}$ What is A max?

Since $a(t) = -(A\omega^2)\cos(\omega t + \phi)$ $a_{max} = A\omega^2 = A(2\pi f)^2$
since $\omega = 2\pi f$

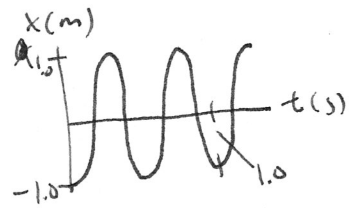
$$a_{max} = 4\pi^2 A f^2$$

$$\therefore\ A_{max} = \frac{a_{max}}{4\pi^2 f^2} = \frac{6.22 \times 10^3\ \frac{m}{s^2}}{4\pi^2 (32{,}768\ Hz)^2} = 1.47 \times 10^{-7}\ \frac{\frac{m}{s^2}}{\frac{1}{s^2}} = 1.47 \times 10^{-7}\ m$$

$$\boxed{A_{max} = 147\ nm}$$

(15.) Analyzing SHM (video)

A. Find $x_1(t)$ given plot



$$x_1(t) = A\cos(\omega t + \phi)$$

Clearly, $A = 1.0\ m$. It starts at its maximum negative value, so $\phi = \pi$
(which we can incorporate by letting $\cos(\omega t + \pi) \rightarrow -\cos \omega t$)

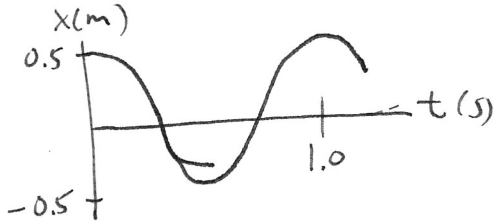
The period is 0.5 seconds, so $f = 2\ Hz$ & $\omega = 2\pi f = 4\pi\ s^{-1}$

So $x_1(t) = (1.00)\cos(4\pi t + \pi)$

or $x_1(t) = -(1.00)\cos(4\pi t)$ where appropriate MKS units are assumed

$$\boxed{x_1(t) = -(1.00\,m)\cos\left[\left(4\pi\ s^{-1}\right)t\right]}$$

(15) B. What is $X_2(t)$ given graph:



Clearly, $A = 0.5\,m$

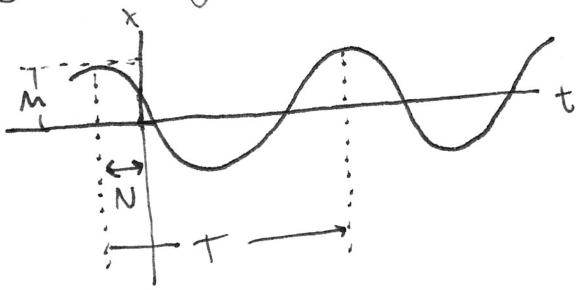
$T = 1.0\,s$

Looks like an "unshifted" cosine function. $\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow \omega = \frac{2\pi}{1s} = 2\pi s^{-1}$

$$\boxed{X_2(t) = 0.5\,m \cos\left[\left(2\pi s^{-1}\right)t\right]}$$

(16) Cosine wave $X(t) = A\cos(\omega t + \phi)$

Given this graph...



A) What's A in the equation?

$$\boxed{A = M}$$

B) What is ω in the equation

$$\boxed{\omega = \frac{2\pi}{T}}$$

C) What is ϕ in the equation?

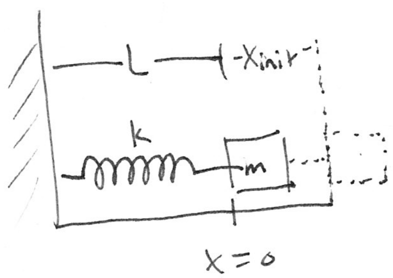
The cosine has shifted left by time = N. This is some fraction of a period; this fraction is $\frac{N}{T}$. One full period of shift would be 2π; so the phase is 2π times the fraction $\phi = 2\pi \frac{N}{T}$. Finally, consider the sign. When ϕ is positive, the cosine shifts left; so we want the positive answer

$$\boxed{\phi = \frac{2\pi N}{T}}$$

(17) HO Kinematics

Here we're working with a general solution that, rather than including a phase term, uses a weighted sum of sine & cosine to model the effect of phase. So

$$x(t) = C \cos \omega t + S \sin \omega t$$



Release block from rest at $x = x_{init}$

A. Find an expression for x_{init}

$x = C\cos(\omega t) + S\sin(\omega t)$ and x_{init} is $x(t=0)$.

In general, $x = C\cos(\omega t) + S\sin(\omega t)$

So $x_{init} = C \underbrace{\cos(0)}_{\to 1} + S \underbrace{\sin(0)}_{\to 0} = C \quad \boxed{x_{init} = C}$

B. Find value of S given that $v(t=0) = 0$

Since $x = C\cos(\omega t) + S\sin(\omega t)$

$v = \frac{dx}{dt} = -\omega C\sin(\omega t) + \omega S\cos(\omega t)$

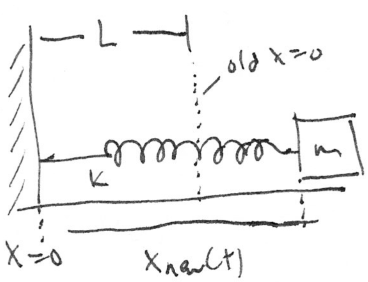
At $t=0$, $v = -\omega C \underbrace{\sin(0)}_{\to 0} + \omega S \underbrace{\cos(0)}_{\to 1} = \omega S$ but $v(0)=0$

So $\omega S = 0 \Rightarrow \boxed{S = 0}$

C. Since $C = x_{init}$ & $S = 0$ we get $\boxed{x(t) = x_{init}\cos(\omega t)}$

D. Find $x_{new}(t)$ for new coordinate system w/ $x=0$ at the wall

We just add L to the old value of x.



$$\boxed{x_{new}(t) = x_{init}\cos(\omega t) + L}$$

(18) Prob. 13.22 simple pendulum

$T_E = 1.75\,s$. On planet X, $T_x = 2.14\,s$. What is g on Planet X? (g_x)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T_E}{T_x} = \frac{2\pi\sqrt{\frac{L}{g_E}}}{2\pi\sqrt{\frac{L}{g_x}}} = \sqrt{\frac{g_x}{g_E}}$$

$$\sqrt{\frac{g_x}{g_E}} = \frac{T_E}{T_x} \rightarrow \frac{g_x}{g_E} = \left(\frac{T_E}{T_x}\right)^2 \rightarrow g_x = g_E\left(\frac{T_E}{T_x}\right)^2$$

$$g_x = 9.8\,\frac{m}{s^2}\left(\frac{1.75\,s}{2.14\,s}\right)^2 = \boxed{6.55\,\frac{m}{s^2}}$$

(19) Ex 13.37

$$x(t) = Ae^{-\frac{bt}{2m}}\cos(\omega t + \phi)$$

If $\frac{b}{2m} = 2.8\,s^{-1}$ how long will it take for amplitude to be half original value

$$\frac{x(t)}{x(0)} = \frac{1}{2} = \frac{Ae^{-\frac{bt}{2m}}}{Ae^{0}}$$

(ignore sinusoidal part; we're just interested in the "envelope" function here

$$\frac{1}{2} = e^{-\frac{b}{2m}t}$$

Take ln of each side

$$\ln\frac{1}{2} = -\frac{b}{2m}t \qquad t = \frac{\ln\frac{1}{2}}{-\left(\frac{b}{2m}\right)} = \frac{\ln\frac{1}{2}}{-2.8(s^{-1})} = \boxed{0.25\,s}$$

(20.) Problem 13.86

A. Rewrite $I \frac{d^2\theta}{dt^2} = -mgL\sin\theta$ for mass on end of massless rigid rod

$I = mL^2 \rightarrow \cancel{m}L^{\cancel{2}} \frac{d^2\theta}{dt^2} = -\cancel{m}g\cancel{L}\sin\theta$

$$\boxed{\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta}$$

B. Describe motion for $K_0 \ll U_{max}$ (i.e. small angle oscillation)

→ oscillatory w/ almost same period as small-angle approximation

C. How does solution "look" compared to small-angle approx. for $K_0 = 0.01\,U_{max}$

→ red & black lines basically on top of one another

D. Describe solution for $K_0 < U_{max}$ (but comparable in magnitude)

→ oscillatory w/ longer period than small-angle formula

E. Pick solution for $K_0 = 0.4\,U_{max}$ (large angle, $< 90°$)

→ oscillates at noticeably slower period than small-angle approx (red "slower" than black)

F. Describe solution for $K_0 > U_{max}$

→ nonuniform motion w/ slow speed at top, fast at bottom

G. ~~Des~~ Pick solution for $K_0 = 1.1\,U_{max}$

→ red "wiggling" but rising without limit