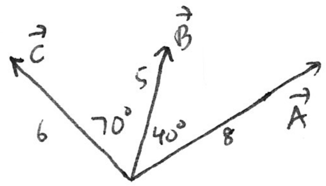
1. Conc. Question 11.01

If two vectors are $\perp$ to each other, their cross product must be zero

FALSE $|\vec{A}\times\vec{B}| = AB\sin\theta$ & $\sin\theta = 1$ for $\theta = 90°$. It cannot be zero unless $\vec{A} = 0$ or $\vec{B} = 0$.

On the other hand, $\vec{A}\cdot\vec{B} = AB\cos\theta = 0$ since $\cos 90° = 0$

2.

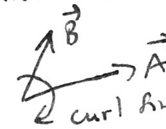


problem 11.06

Find $\vec{B}\times\vec{A}$

$|\vec{B}\times\vec{A}| = BA\sin\theta = (5)(8)\sin 40° = 25.7$

Direction by right-hand rule...



curl fingers of right hand CW → into page

so $\boxed{\text{"26, directed into the plane"}}$

3. Vector Cross Product

$\vec{A} = (1, 0, -3)$

$\vec{B} = (-2, 5, 1)$

$\vec{C} = (3, 1, 1)$

A. $\vec{B}\times\vec{C} = \hat{i}(5\cdot1 - 1\cdot1) + \hat{j}(1\cdot3 - (-2)\cdot1) + \hat{k}(-2\cdot1 - 5\cdot3)$

$= 4\hat{i} + 5\hat{j} - 17\hat{k} = \boxed{(4, 5, -17)}$

B. $\vec{C}\times\vec{B} = -\vec{B}\times\vec{C} = -4\hat{i} - 5\hat{j} + 17\hat{k} = \boxed{(-4, -5, 17)}$

C. $(2\vec{B})\times(3\vec{C}) = (-4, 10, 2)\times(9, 3, 3) = (2\cdot3)(\vec{B}\times\vec{C}) = (6\cdot4, 6\cdot5, 6\cdot(-17))$

$(2\vec{B}\times3\vec{C}) = \boxed{(24, 30, -102)}$

D. $\vec{A}\times(\vec{B}\times\vec{C}) = (1, 0, -3)\times(4, 5, -17) = \hat{i}(0\cdot(-17) - (-3)(5)) + \hat{j}((-3)(4) - (1)(-17)) + \hat{k}((1)(5) - (0)(4))$

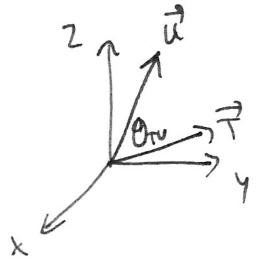
$= 15\hat{i} + 5\hat{j} + 5\hat{k} = \boxed{(15, 5, 5)}$

E. $\vec{A}\cdot\vec{B}\times\vec{C} = (1, 0, -3)\cdot(4, 5, -17) = 1\cdot4 + 0\cdot5 + (-3)\cdot(-17) = 4 + 0 + 51 = \boxed{55}$

F. If $\vec{V}_1 \perp \vec{V}_2$, $|\vec{V}_1\times\vec{V}_2| = V_1 V_2 \sin 90° = \boxed{V_1 V_2}$

G. If $\vec{V}_1 \parallel \vec{V}_2$, $\theta = 0°$ so $|\vec{V}_1\times\vec{V}_2| = V_1 V_2 \sin 0° = \boxed{0}$

4. Finding the Cross Product



A. Express $\vec{V}$ as an ordered triplet, separated by commas

$\vec{V} = \vec{T} \times \vec{U} = \hat{\imath}\left((1)(0) - (0)(4)\right) + \hat{\jmath}\left((0)(2) - (3)(0)\right) + \hat{k}\left(3 \cdot 4 - 1 \cdot 2\right) = 10\hat{k}$

$\rightarrow \boxed{\vec{V} = (0, 0, 10)}$

$\vec{T} = (3, 1, 0)$

$\vec{U} = (2, 4, 0)$

$\vec{T} \times \vec{U} = \vec{V}$

B. $\boxed{|\vec{V}| = 10}$ $\left(= \sqrt{V_x^2 + V_y^2 + V_z^2} = \sqrt{0^2 + 0^2 + 10^2} = 10\right)$

C. Since $|\vec{T} \times \vec{U}| = |\vec{T}||\vec{U}| \sin\theta$,

$V = TU \sin\theta \rightarrow \sin\theta = \frac{V}{TU}$ & $\theta = \sin^{-1}\frac{V}{TU}$

We need T & U: $\left.\begin{array}{l} T = \sqrt{3^2 + 1^2 + 0^2} = \sqrt{10} \\ U = \sqrt{2^2 + 4^2 + 0^2} = \sqrt{20} \end{array}\right\} \Rightarrow \theta = \sin^{-1}\frac{10}{\sqrt{10}\sqrt{20}} = \sin^{-1}\frac{10}{\sqrt{200}}$

$\theta = 45°$

Question asks for $\sin\theta = \frac{10}{\sqrt{200}} = \frac{1}{\sqrt{2}}\frac{10}{\sqrt{100}} = \boxed{\frac{1}{\sqrt{2}} = 0.707}$

5. Spinning the Wheels

A. Since $\vec{L} = \vec{r} \times \vec{p}$

$= \vec{r} \times m\vec{v}$, using the units of $r, m, v \rightarrow m \cdot kg \cdot \frac{m}{s} \rightarrow \boxed{\frac{kg\,m^2}{s}}$

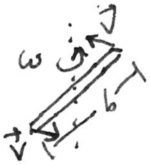
B. Given I & α, what is L after time t if it starts from rest? For constant α,

First, we need rotational kinematics to get ω at time t.

$\omega = \cancel{\omega_0}^{0} + \alpha t \rightarrow \omega = \alpha t$

Now use $L = I\omega \rightarrow \boxed{L = I\alpha t}$

C. A rigid uniform bar, mass m, length b, rotates about mid point. End points of bar have speed v. What is L?



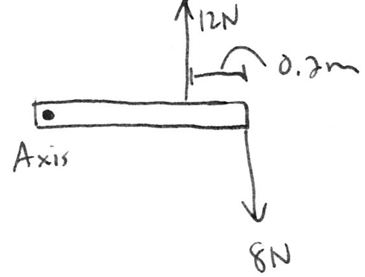
First, we'll use $L = I\omega$ for a rigid body. We need both I & ω from given info. For I, Table 10.2 gives $I = \frac{1}{12} ML^2$ for a rod of length L.

So we have $I = \frac{1}{12} mb^2$

For ω, note that $v = r\omega$. In this case, $r = \frac{b}{2}$ so $v = \frac{\omega b}{2} \rightarrow \omega = \frac{2v}{b}$

So $L = \left(\frac{1}{12} m b^{\cancel{2}}\right)\left(\frac{2v}{\cancel{b}}\right) = \boxed{\frac{mbv}{6}}$

5 D.



The bar has a length 0.80m. It begins to rotate from rest. What is $\vec{L}$ after 6.0s?

Need torques & rotational N2L $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$. For constant $\vec{\tau}$, $\vec{\tau}\Delta t = \Delta\vec{L}$ and since $\vec{L}_0 = 0$, we get $\vec{L} = \vec{\tau}\Delta t$. We need only worry about magnitudes given the fixed axis of rotation

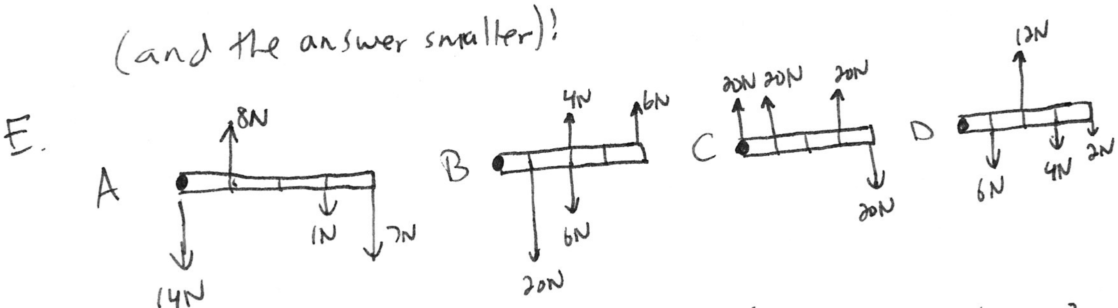
τ_{Net}: The 12N force provides a (+) torque while the 8N force provides a (−) torque. Given the total length of the bar (0.80m), the 12N force acts at a distance (0.80m − 0.20m) = 0.6m. So

$$\tau_{Net} = \tau_{12N} + \tau_{8N} = (0.6m)(12N) - (0.8m)(8N) = 0.8 N\cdot m$$

$$L = \tau\Delta t = (0.8 N\cdot m)(6.0s) = 4.8\ N\cdot m\cdot s = 4.8\ \frac{kg\,m}{s^{\cancel{2}}}\cdot m\cdot \cancel{s} = \boxed{4.8\ \frac{kg\,m^2}{s}}$$

Note that we assume the two forces change directions so as to remain ⊥ to the bar throughout this motion. Otherwise the problem is a lot harder (and the answer smaller)!

E.



For which is $\tau_{net} = 0$? $\tau_A = (8N)\left(\frac{L}{4}\right) - (1N)\left(\frac{3L}{4}\right) - (7N)L = L\left(2N - \frac{3}{4}N - 7N\right) = -5\frac{3}{4}N\cdot L \neq 0$

$\tau_B = (-20N)\frac{L}{4} - (2N)\frac{L}{2} + 6N L = (-5N - 1N + 6N)L = 0$ ✓

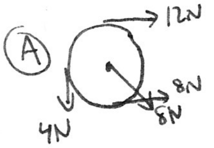
$\boxed{B, C} \Leftarrow$ $\tau_C = 0 + 20N\left(\frac{L}{4}\right) + 20N\frac{3L}{4} - 20NL = (5N + 15N - 20N)L = 0$ ✓

$\tau_D = -6N\left(\frac{L}{4}\right) + 12N\left(\frac{L}{2}\right) - 4N\left(\frac{3L}{4}\right) - 2NL = (-1.5N + 6N - 3N - 2N)L \neq 0$
$= -0.5NL$

F. For which diagrams is $\vec{L}$ constant?
Since $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$, it's the ones where $\tau_{net} = 0$: $\boxed{B, C}$

5. G.

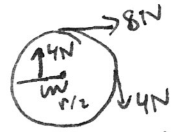
(A)



τ_{net}: $(-12N)r + (8N)r + (4N)r + 8N(0)$

$\tau_{net} = 0$

(B)

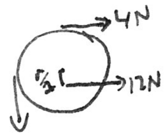


$(-8N)r + -(4N)\frac{r}{2} - (4N)r$

$= (-14N)r$

$\neq 0$

(C)

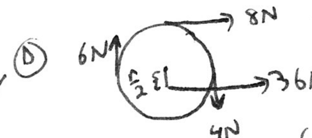


$-(4N)r + 8N(r) + \frac{r}{2}(12N)$

$= (10N)r$

$\neq 0$

(D)



$-(8N)r - (4N)r - 6N(r) + (36N)\frac{r}{2}$

$= -(18N)r + (18N)r = 0$

$\tau_{net} = 0$

For which diagrams is $\vec{L}$ constant?

Since $\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$, $\vec{L}$ = const if $\tau_{net} = 0$. So calculate τ_{net} for each

(see diagrams) $\rightarrow$ $\boxed{A, D}$

H. 3 disks spin & collide along axis. Initial conditions are

Disk 1	I	ω (CW)	$\rightarrow L_1 = I(-\omega) = -I\omega$
Disk 2	$2I$	3ω (CCW)	$\rightarrow L_2 = 2I(+3\omega) = +6I\omega$
Disk 3	$4I$	$\frac{\omega}{2}$ (CW)	$\rightarrow L_3 = 4I(-\frac{\omega}{2}) = -2I\omega$

$L_i = L_1 + L_2 + L_3 = -I\omega + 6I\omega - 2I\omega = 3I\omega$

Since there are no net torques exerted from outside the system, $L_f = L_i = 3I\omega$

The final, total value of rotational inertia is the sum of the rotational inertias of the disks: $I_{tot} = I_1 + I_2 + I_3 = I + 2I + 4I = 7I$

Therefore

$I_f = I_{tot}\,\omega_{net} = 3I\omega$

$7\cancel{I}\omega_{net} = 3\cancel{I}\omega$

$\boxed{\omega_{net} = \frac{3}{7}\omega}$

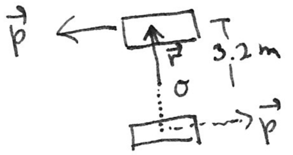
6. Problem 11.37 Two 1800 kg cars traveling at 95 $\frac{km}{h}$ in opposite directions move in straight lines. The CM of each car is 3.2 m from the center of the roadway

A. What is L_{total}?

First, note that each car has the same $\vec{L}$. Choose the system center of mass as the origin and, for simplicity, calculate $\vec{L}$ at the moment when the cars pass one another. Because there are no net torques on the cars, whatever value of $\vec{L}$ we calculate at this location is unchanged for times before & after they pass.

For 1 car, $|\vec{L}| = |\vec{r} \times \vec{p}| = |\vec{r} \times (m\vec{v})| = mvr\sin\theta$ ($\theta = 90°$)

$L = mvr$

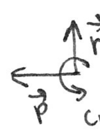


$$L = (1800\,kg)\left(95\,\frac{km}{h}\right)\left(\frac{1h}{3600s}\right)\left(\frac{1000m}{1km}\right)(3.2m)$$

$$= 152{,}000\ kg\frac{m^2}{s} = 152{,}000\ J\cdot s$$

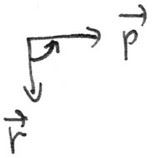
Note that the right-hand rule (RHR) says $\vec{L}$ points out of the page

For the "top" car $\vec{r}, \vec{p}$ set tail-to-tail:



curl fingers of right hand; thumb is up

Same result for other car:



So the sum is going to give twice L of one car (rather than zero). You need to answer (B) before (A), in fact!

So $L_{total} = 2L = \boxed{300{,}000\ J\cdot s}$

B. Direction - $\boxed{\text{Same for both, out of plane of image}}$

7. Drop a long, heavy bean bag from above into lap of man spinning freely on a stool. What happens?

$\boxed{\text{He spins slower}}$ $\vec{L}$ is conserved, so $I_1\omega_1 = I_2\omega_2$. Adding the bean bag increases I, so $I_2 > I_1$.

This means $\omega_2 = \frac{I_1}{I_2}\omega_1 < \omega_1$ since $I_1 < I_2$

8. Two identical pucks collide & rotate about their cm at ω_1 after a completely inelastic collision

Before After

Suppose we double mass of each. How is new collision different? They will rotate $\boxed{\text{at the same rate}}$ (as before).

Why? Using $\vec{L} = \vec{r}\times\vec{p} = \vec{r}\times m\vec{v}$, doubling each m doubles $\vec{L}$.

But for the final system, doubling m also doubles I.

For the original collision, $L_{tot} = mvr + mvr = 2mvr = I\omega_1$

" " second " , $L_{tot} = (2mvr) + (2mvr) = \underbrace{4mvr = 2I\omega_2}$

dividing both sides by 2 gives same formula

$$\omega_1 = \frac{2mvr}{I}$$

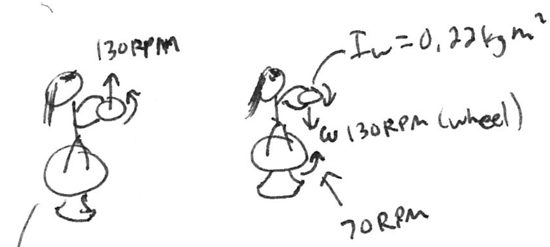
$$\omega_2 = \frac{4mvr}{2I} = \frac{2mvr}{I}$$

Equal!

9. Problem 11.44

A. Find student's mass. Model her as a solid cylinder, $d = 30\text{cm}$

Before | After



$I_{tt} = 0.31\,kgm^2$

First, Table 10.2 for a cylinder rotating about its axis gives $I_G = \frac{1}{2}MR^2$ M = unknown, $R = 15\text{cm}$

Apply conservation of $\vec{L}$ (turntable ensures no net external torques on system)

$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_{tt_i} + \vec{L}_{w_i} = \vec{L}_{tt_f} + \vec{L}_{w_f}$$

Assume $L_{wf} = L_{wi}$ & take upward as the positive direction for $\vec{L}$

$$\cancel{L_{tt_i}}^{\,0} + L_{w_i} = L_{tt_f} + L_{w_f}$$

$$I_w\omega_0 = (I_{tt} + I_G)\omega_{tt} - I_w\omega_0$$ ← Assume wheel does not slow as she flips it

$$2 I_w\omega_0 = (I_{tt} + I_g)\,\omega_{tt}$$

$$I_{tt} + I_g = 2 I_w \frac{\omega_0}{\omega_{tt}}$$

$$I_g = 2 I_w \frac{\omega_0}{\omega_{tt}} - I_t = \frac{1}{2}MR^2$$

$$MR^2 = 4 I_w \frac{\omega_0}{\omega_{tt}} - 2 I_{tt}$$

$$M = \frac{4I}{R^2}\frac{\omega_0}{\omega_{tt}} - 2\frac{I_{tt}}{R^2} = \frac{4(0.22\,kgm^2)}{(0.15m)^2}\left(\frac{130RPM}{70RPM}\right) - 2\,\frac{0.31\,kgm^2}{(0.15m)^2}$$

$$\boxed{M = 45\,kg}$$

B. What work did she do flipping the wheel? For this, just find ΔK_{rot}

Need ω_{tt} in $\frac{rad}{s}$

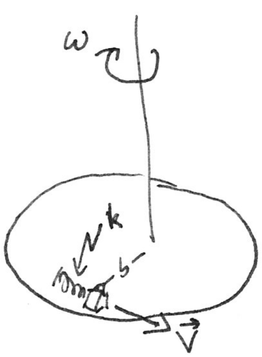
$$W = \Delta K_{rot} = \underbrace{\left[\cancel{\frac{1}{2} I_w \omega_w^2} + \frac{1}{2}(I_{tt} + I_G)\,\omega_{tt}^2\right]}_{K_f} - \underbrace{\cancel{\frac{1}{2} I_w \omega_w^2}}_{K_i}$$

$$= \frac{1}{2}\left(0.31\,kgm^2 + \frac{1}{2}(45kg)(0.15m)^2\right)\left[\left(70\,\frac{rev}{min}\right)\left(\frac{1\,min}{60s}\right)\cdot\left(\frac{2\pi\,rad}{1\,rev}\right)\right]^2$$

$$\boxed{W = 22\,J}$$

10. Problem 11.58

massless spring launches block of mass m on frictionless turntable of rotational inertia I (originally compressed a distance x)



A. Find v   B. Find ω   after the mass is launched

Use conservation of mechanical energy $\Delta K + \Delta U = 0$

AND

conservation of angular momentum $L_i = L_f$

Cons of energy: $\Delta K + \Delta U = 0 \rightarrow \left(\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$

cons of $\vec{L}$: $0 = \vec{r}\times\vec{p} + I\vec{\omega}$ $\underbrace{r = b,\ \vec{p} = m\vec{v},\ \vec{r}\perp\vec{v}}_{\text{block } \vec{L} \text{ is in } +z \text{ direction}} \Rightarrow |\vec{r}\times\vec{p}| = mvb$. Since $\vec{\omega}$ is down, $\vec{\omega} = -\omega\hat{z}$

↳ $I\omega = mvb$

Eliminate ω first → $\omega = \frac{mvb}{I}$ sub into energy equation

$$\cancel{\frac{1}{2}}\cancel{I}\overset{\omega}{\left(\frac{mvb}{\cancel{I}}\right)^2} + \cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}kx^2$$

$$\frac{m^2v^2b^2}{I} + mv^2 = kx^2$$

$$\left(\frac{m^2b^2 + Im}{I}\right)v^2 = kx^2$$

$$v^2 = \frac{Ikx^2}{m^2b^2 + Im} \rightarrow \boxed{v = \sqrt{\frac{Ikx^2}{m^2b^2 + Im}}}$$ part A

B: $$\omega = \frac{mvb}{I} = \frac{mb}{I}\sqrt{\frac{Ikx^2}{m^2b^2 + Im}} = \sqrt{\frac{m^2b^2\cancel{I}kx^2}{I^2m^2b^2 + I^3m}} = \sqrt{\frac{mb^2kx^2}{Imb^2 + I^2}}$$

11. A 3.2 m diameter merry-go-round with $I = 140\ kg\,m^2$ spins at $0.50\ \frac{rev}{s}$. 4 25-kg children sit suddenly on the edge.

A. Find new ω.

Apply conservation of $\vec{L}$: $I_1 \omega_1 = I_2 \omega_2 \rightarrow \omega_2 = \frac{I_1}{I_2}\omega_1$

$$\left.\begin{array}{l} I_1 = I_{tt} = 140\,kgm^2 \\ I_2 = I_{tt} + 4 I_{child} \\ I_{child} = Mr^2 \\ r = \frac{3.2m}{2} = 1.6m \end{array}\right\} \Rightarrow \omega_2 = \frac{I_{tt}}{I_{tt}+4I_c}\omega_1$$

$$= \left(\frac{140\,\cancel{kgm^2}}{140\,\cancel{kg}\,\cancel{m^2} + 4(25\cancel{kg})(1.6\cancel{m})^2}\right) 0.5\frac{rev}{s}$$

$$\boxed{\omega_2 = 0.18\ \frac{rev}{s}}$$

B. Energy lost to friction

Find ΔK_{rot} & set difference equal to frictional energy loss

ω must be in $\frac{rad}{s}$

$$\Delta K = \frac{1}{2} I_2 \omega_2^2 - \frac{1}{2} I_1 \omega_1^2 = \frac{1}{2}(I_{tt} + 4I_c)\omega_2^2 - \frac{1}{2} I_{tt}\omega_1^2$$

$$= \frac{1}{2}\left(140\,kg\,m^2 + 4(25kg)(1.6m)^2\right)\left(0.18\frac{rev}{s}\cdot\frac{2\pi\,rad}{1\,rev}\right)^2 - \frac{1}{2}(140\,kgm^2)\left(0.5\frac{rev}{s}\cdot\frac{2\pi\,rad}{1\,rev}\right)^2$$

$$= \frac{1}{2}(140\,kgm^2 + 256\,kgm^2)\left(1.13\frac{rad}{s}\right)^2 - \frac{1}{2}(140\,kgm^2)\left(3.14\frac{rad}{s}\right)^2$$

$$= 253\,J - 691\,J = \boxed{438\,J}$$

(450 J in Mastering Physics because I rounded ω_f too much)