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Perception of Form in Biological and Man-Made Systems

Heinz Von Foerster

(1962)

Heinz Von Foerster, native of Vienna, served as a physicist in leading research laboratories in Germany before and during World War II. Later, he was Editor in Chief for Science and Art with the Radio Network of the Information Service Branch, U.S. Army in Vienna. Von Foerster was a Guggenheim Fellow in 1956, consultant to U.S. Department of Health, Education and Welfare from 1957 to date. Presently he is Director of the Biological Computer Laboratory and a Professor in the Department of Electrical Engineering at the University of Illinois. Editor's Note: This lecture was originally given at an annual meeting of the Industrial Design Education Association, held at the University of Illinois on March 17, 1962. As such, it has elicited so much response that we are still receiving requests for copies. We are taking this opportunity to reprint it in its entirety for the benefit of those who have not had the opportunity to read it.

The choice of the topic of my presentation this morning is due to the fact that in two fields, in the field of the neurophysiology of perception and in the mathematical and topological formulation of "form" tremendous progress has been achieved in the last couple of years. The instrument of progress in neurophysiology of perception is a small gadget: the "microprobe". It is an extraordinarily delicate electric probe, capable of recording the electric activity of single nerve fibers, and it enables us to learn what these fibers are doing. Furthermore, we can now electrically excite specific regions of the brain and obtain a verbal record of the conscious person who is asked what he sees, feels, or hears, while these regions are excited. Thus we are finally in a position to formulate theories of perception and to check them by probing into different regions of the brain, seeing whether these theories have any validity at all.

On the other hand, a mathematical theory of form has been reasonably well established through the developments in group theory, topology, set theory and other special topics in mathematics, and you will allow me to give just a few examples in these fields.

At first I would like to establish certain relationships between the two concepts "form", and "perception". Both these concepts deal with a process which is well understood and generally referred to as "abstraction". Take as an example this cubical box. I turn it around in different directions; you will always see different projections of this cube on your retina, but in each case you can say, "He has a cubical box in his hand. This box is not going to change." It stays the same box in your head, despite the fact that you see always something different. There is obviously something constant in all these gyrations: the object's "cubicality". This is the entity which does not vary. Mathematically, the entities which do not vary are called "invariants". Form does not vary, hence it is an invariant. Our language has solved the problem beautifully. We give invariable[s] names, as e.g., "cube" to these invariants. By naming it we have immediately defined the invariability of the form of the entity in my hand and call[ed] it a cube. If the object of my example would have constantly changed into completely different shapes we would call it perhaps an amoeba. However, in spite of the fact that an amoeba is constantly changing its shape it still has a name, because there are certain things about an amoeba that are invariant — certainly not the shape, but perhaps the volume or the rates of changes of motion or some other more abstract properties.

Having established the concept of an "invariant", let me now show what is meant by an abstraction. The process of abstraction is nothing else but the computation of some invariants. Let me give you a simple example. Consider that you keep your head rigid and move just your eye-balls back and forth from left to right. Despite the fact that the image of this room falls on different areas of the retina, the room remains stationary.

Now, let's reverse the observation. We excite a small region in the retina, say with an electric probe. Or, to mention a more unpleasant way of excitation, someone knocks you over the head. An unintentional aesthetic side-effect happens: one sees stars. These stars are produced by a local irritation of the retina and as a consequence there are certain shapes and brightnesses transmitted to the brain. If you then move your eyeballs, the peculiar thing is that these stars or shapes are zooming about with the motion of your eyeball, in spite of the fact that always the same spot of the retina is being excited.

Note that in the first case *different fields* of the retina are being excited and one has the impression of stationarity. In the second case, although only *one spot* on the retina is excited one has the feeling the stimulus comes from different directions.

If one probes deeper into the brain, into the lateral geniculate[1] region, (a deep region in the brain after the schizma of the optical nerve) and excites a small spot in this region, you will see a well defined light spot in a particular direction, but it will be stationary. You may move your eyes around, [but] the spot will stay put, independent of the motions of your eyes. Obviously what is done in this region is that a certain invariant is computed which gives you information independent of a particular bit of information, namely direction. If one goes further along the optic nerve and excited regions one level higher up, (which would correspond to the lower region of the visual cortex) the interesting thing is that you see certain bright shapes, but you don't know in which direction they are. A patient so excited sees a certain spot. Asked to move the eyes around and to report the direction of the spot, the patient says, "Well, I don't know in which direction it is, I only see a bright spot of a particular shape." — but directionality is gone. Obviously we have passed a computer which gets rid of particular directional feelings and fixes ones attention to information about brightness and shape. If one goes still further and excites the higher levels of the visual cortex, one may get only the answer, "I see light." No shape, no form, no direction, independent of position of head or eyes. In other instances one may elicit the appearance of very intricate shapes, but size, illumination and direction is gone.

I hope that these few examples suggest the kind of processes we have to look for, if we want to get some insight into the problem of how form is perceived. Clearly these processes are carrying out abstractions, or — to put it into mathematical jargon — these processes compute invariants on the visual image that is thrown on the retina which serves — so to say — as the first relay in a whole chain of abstractors. From a neurophysiological point of view we have thus to look for a chain of computers which are capable of computing higher and higher abstractions or invariants.

I have now translated a somewhat unyieldy problem, namely, that of perception of form into tangible, neurophysiological and mathematical concepts, because talking about perception of form is equivalent to talking about the computation of invariants. However, it is my personal experience that in the moment when one talks about mathematical concepts in an artistically oriented group, everybody locks himself up behind the conviction: "Mathematics and I are not made for each other".

Ladies and Gentlemen, please, take it easy. First, I promise you that today I shall not use a single equation, nor a single number in my further presentation, except a few numbers which are so exceptionally large that even mathmaticians have to confess that they are unable to comprehend their magnitude. It is perhaps the artist who is able to get a sense of feeling for them. I shall need these numbers later to support some of my statements regarding the complexity of some of the concepts involved.

Second, if you feel for a moment that you do not follow what I'm trying to present to you, always keep in mind that there is a one-to-one correspondence between the problem of computation of invariants and the other problem, which I believe you understand very well, namely, [the] perception of form.

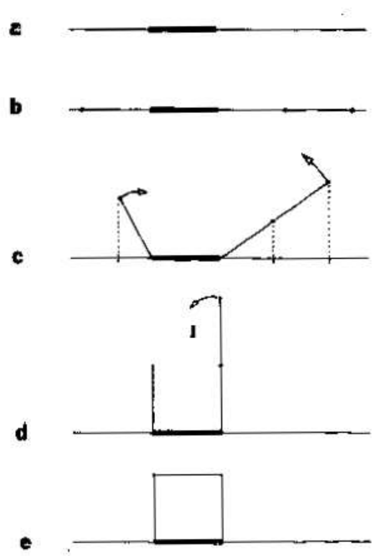
I shall now proceed in tackling my topic of perception of form by developing my theme in three steps. First, I shall elaborate further on the concept of an invariant by pointing out that not only certain geometric relationships as, e.g., angles, number of planes, number of corners, etc., may define invariants, but also the principles of the generation of these relationships may be invariant. Second, I shall try to show how these invariants are computed by our nervous system and how we may use this knowledge to build electronic systems which embody these biological principles. Finally, I shall return to my original topic and shall demonstrate on a few examples the significance of certain invariances in the perception of form.

Let me go back for a moment to the cardboard cube I have just shown to you in various positions. I am convinced that you will not argue with me, if — for the sake of generality — I call a square, which is a figure in two-dimensions, a "two-dimensional cube". I could, of course, call a cube a "three-dimensional square", but let's stick to the generic name "cube" or, more precisely "N-dimensional cube" for all closed N-dimensional geometrical figures which are composed of (N-1)-dimensional cubes joined together in right angles. Clearly, this definition holds for our familiar three-dimensional cube, because it is a closed geometrical figure composed of two-dimensional cubes (= squares) which are joined on the edges under right angles. However, I would like to draw your attention to the peculiar way in which I defined an "N-dimensional cube" namely by using in my definition a cube of (N-1) dimensions. You may protest and say that it is unfair — if not even cheating — to use the definiendum[2] in the definition. But I did not do this, for a cube of N dimensions is certainly not a cube of (N-1) dimensions. On the other hand, this definition illuminates two points I would like to make. First, it gives us a prescription of how to construct cubes of N dimensions, if we know a cube of N-1 dimensions. Second, this definition is invariant to the dimensionality of the space in which

[1] *geniculate*: abruptly bent.

[2] *definiendum*: That which is, or is to be, defined.

we would like to construct our cube.

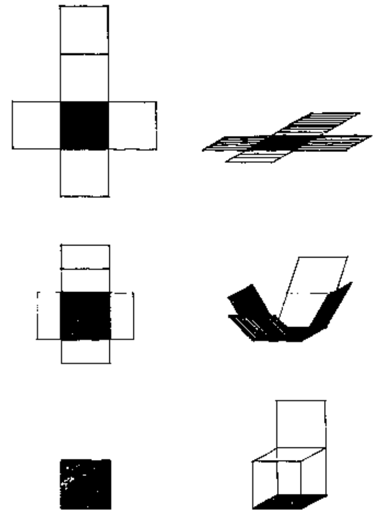


1. *Generation of a "two-dimensional cube" by folding 3 "one-dimensional cubes" into two-space.*

Utilizing the power of my recursive definition I shall now proceed in constructing a four-dimensional cube by starting with a one-dimensional cube. Fig. (1a) shows you such a cube, which you would probably call a "line-segment". You note that this "cube" is a geometrical figure of one dimension (N=1), closed by two zero-dimensional cubes (points). We now proceed to build a two-dimensional cube out of one-dimensional cubes. To this purpose we assemble to all "sides" of our one-dimensional cube other one-dimensional cubes which we intend to tilt around the sides of our centerpiece until they will stick out perpendicularly into two-space. The plane has two dimensions, hence it is a "space" with two dimensions or, short "two-space". Ordinary space is "three-space", and so on.

Since we are farsighted builders, we realize that we need, besides two one-dimensional cubes on both edges, another one-dimensional cube (Fig. 1b) which will serve as a lid in order to produce as a final result a closed geometrical figure. Having everything prepared, we start now our tilting operation (1c) until 90°are reached, and finally we close the figure by closing the lid (1d), and a two-dimensional cube leaves the production line.

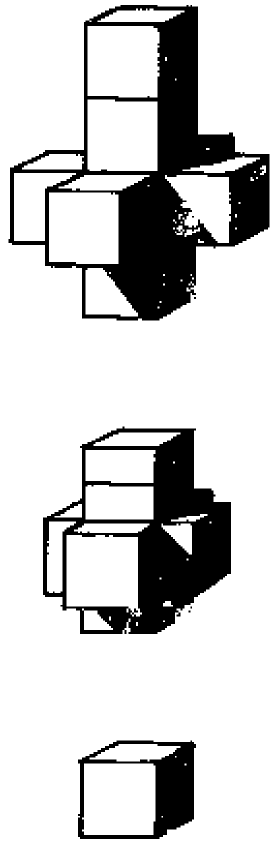
I hope you will pardon my involved telling of an otherwise trivial story. But try for a moment to put yourselves into the shoes of, say, one-dimensional people who live in the one-dimensional world of the line in which we assembled the parts for the two-dimensional cube. In the moment when the tilting operation begins, for these people the tilted segments will disappear into "unfathomable dimensions", the only thing they may see is the projections of these segments into their space of comprehension, and the only guarantee they have that the tilting operation progresses and finally succeeds is that these projections become shorter and shorter until they shrink to a point. At that instant they know the segments are perpendicular.



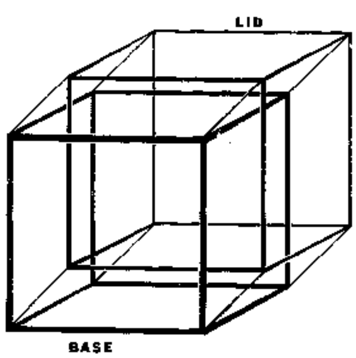
2. *Generation of a three-dimensional cube by folding 5 two-dimensional cubes into three space.*

Fig. 2 repeats this operation for the construction of a three-dimensional cube out of two-dimensional cubes. I am sure, all of you have, at one occasion or another, built such a cardboard cube. We proceed exactly as before, assembling in two-space the appropriate number of two-dimensional cubes around the "centerpiece", not forgetting the "lid", and start the tilting operation. Observers in the plane will see only the projections of the building blocks going up in three-space (Fig. 2 left side). At the instant, when these projections vanish, they know they have succeeded in getting them perpendicular to the centerpiece. Although a normal projection of this three-dimensional entity does not look too interesting — one just sees the black centerpiece — the two dimensional observer may get a kick out of his achievement by projecting his result obliquely into his flatland or by tilting the result and obliquely projecting it (Fig. 2 right side).

With the aid of our recursive definition we proceed now to construct a four-dimensional cube, keeping in mind that our construction principle is an invariant (Fig. 3). Exactly as before we take a "centerpiece", now a three-dimensional cube, and attach to each of the six sides another three-dimensional cube and, of course, do not forget the lid which we put on top of our structure (3a). We are now ready for our tilting operation into four-space around the six planes of the centerpiece. We carefully watch the shrinking of the projections into our three-space, (3b) and stop tilting at the moment these projections vanish (3c). We give the whole structure a little kick, hoping that this will close the lid, and our four-dimensional cube can leave the construction line. Again, a normal projection of this four-dimensional entity does not look too interesting. However, we may get a feeling for the richness of this structure if we obliquely project our result into three-space. Fig. 4 shows such a projection made at about 30°. Center-cube (strong lines) and lid appear as undistorted cubes, because they are parallel to our space. The six other cubes however, being perpendicular to our space, suffer some distortions and appear as rhombohedrons.

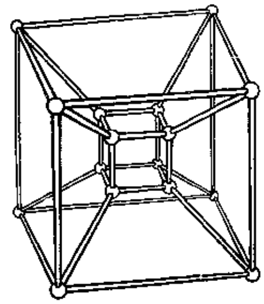


3. *Generation of a four-dimensional cube by folding 7 three-dimensional cubes into four-space.*



4. *Three-dimensional lattice of an obliquely projected four-dimensional cube parallel to our three-space.*

Sometimes a perspective view is helpful. Such a view is given in Fig. 5 which is obtained by drilling a peephole into four-space and looking through this in a close-up at our cube.[1] The center-cube is the lid, the frame is the centerpiece and the side-cubes are the truncated four-sided pyramids connecting the frame with the center cube.



5. *Perspective representation of a four-dimensional cubic lattice seen in four-space.*

It is not difficult to imagine how this presentation would change, if we would tilt this four-dimensional cube in four-space and let the projections go through different angles. Depending upon the angle of projection, you would see the center-cube move about in the frame as if suspended by rubber strings. Tilting the structure in four-space would let frame and center-cube assume elongated or shortened, rhomboidal or parallelepipedic shapes.

I shall spare you the mental gyrations necessary to construct 5, 6, 7 ... etc. dimensional cubes, although, as you have seen, there are no particular problems to be encountered. For a five-dimensional, or 5D, cube we just need again a 4D center-cube and eight 4D side-cubes attached to the center piece. Of course, we should not forget the lid. Thus, ten four-dimensional cubes will constitute a five-dimensional cube, etc., etc. At this point you may say: “And what has all that to do with this topic? Why such extremely abstract examples? We never run into those things!”

You are wrong. We constantly run into those things, and since in many instances we know the construction rules, the invariants, of such entities, we are able to construct the spaces of their residence. Let me illustrate this by three young gentleman[2] you see in Fig. 6. He looks very good; however, there is some strangeness about him. He seems to be “distorted”, “unduly elongated” — with respect to what, may I ask? Of course, with respect to a normally proportioned young gentleman whose construction rules are invariants in our minds. This young gentleman is indeed a normally proportioned human specimen, only that he is sitting in four-space and what you see is his three-dimensional oblique projection. To use a somewhat weak analogy, think of the elongated shadows you see of yourself at grazing angles of the sun. The sculpture may be thought of as the three-dimensional “shadow” of a well proportioned youth in four-space. Your interpretation of this added dimension is immediately invoked by a strange feeling of “unearthliness” or “etherealness”, something you’ll never get out from a reasonable replica of man one sees in display-windows or in the Museum of Natural History.

6. *Three-dimensional image of an oblique projection of a three-dimensional youth of normal proportions tilted into four-space. (Lehmbruck: "Standing Youth"; Mus. Mod. Art, NY)*

The second example is the young lady[3] of Fig. 7. This image of bursting vitality, how can this be explained? With little training in four-dimensional geometry you may be able to prove to yourself that she is the exact complement to our previous example. The entity you see is the actual model in four-space which, projected obliquely into three-space, would produce young ladies of whatever desirable proportions, depending only on the chosen angle of projection. Turning her symmetry-plane more or less into the ray of projection, one may generate Sophia Loren-types or Vogue-models respectively, *chacun à son goût*.[3] Maillot's success in taking this generatrix for so many shapes out of four-space and letting her join us here on earth probably accounts for this sculpture's incredible dynamicism.

In my third and final example I would like to introduce to you a "non-linear" three-dimensional space, i.e., one that has a strong curvature in four-space (Fig. 8). This girl[4] does not look so sad because she has an overgrown hand. She is probably unhappy because her hand and arm sit in a three-dimensional "bubble". The simplest analogy in two-space is a picture made on a rubber sheet which afterwords is locally expanded into three-space.

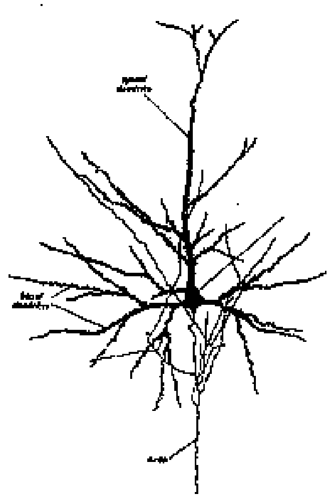
In all my previous examples I have made allusions as to invariants which either were principles of generation of shapes or fixed spatial relationships that are unchanged with respect to variations of dimensionality, size, position, curvatures, projections, etc. However, I have not yet given any clues as to how we — or better — how our nervous system is capable of appreciating these invariants. I said before, these invariants are "computed". Does this mean that our brain is capable of computations which are — in a sense — non-numerical? The answer today is a very definite yes, and I hope I shall be able to show you later that already in the lowest levels of evolution primitive nervous systems are doing just this kind of computation.

7. *Three-dimensional model which when tilted into four-space and projected obliquely, gives a three-dimensional image of normal proportions. (Maillol: "Chained Action"; Mus. Mod. Art, NY).*

8. *Non-linear three-space having developed a "bubble" in four-space. (B. Brandt: "Perspectives sur le Nu", #12).*

However, before I can discuss these physiological computer-systems, I have to give you a brief account of its basic element which is, of course, the nerve-cell, or, for short, the neuron. As you will see in a moment, this cell is in itself a highly complex computer, capable of many more operations than the simple electron-tubes or transistors used in modern high speed electronic computers.

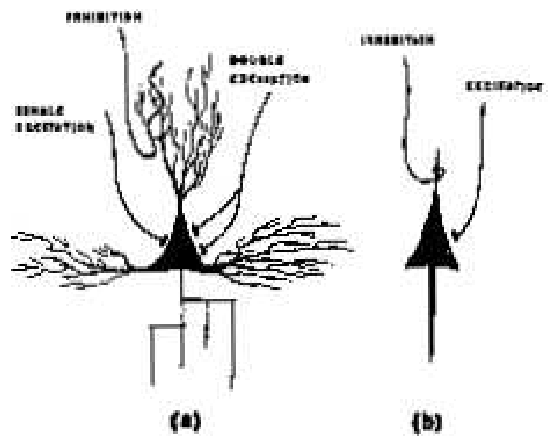
[3] *chacun à son goût*: each one to their taste.



9. *Cell body, dendrites (upwards) and axon (downwards) of a cortical neuron.*

After half a century of intensive study of the neuron, its physiology, topology, chemistry, electro-chemistry, molecular structure, etc., we still cannot say today that we know precisely all the features of this fascinating elementary organ. However, we can say today, that we are in possession of different levels of approximations of its structure as well as of its function, and in the last few years models of these approximations have been realized in electronic "hardware" which opened new vistas for physiologists, psychologists and for the inventive and enterprising engineers.

Figure 9 shows one of the several billion neurons in the outer folds of a cat's brain.[5] The big blop in the center is the cell-body proper, the "soma", which houses the cell's nucleus and is probably responsible for the neuron's metabolic activity. The same membrane which envelopes the soma forms also the tubular sheaths around the many ramifications extending from the soma. There are two kinds. One kind is seen branching off in all directions in a tree-like fashion, the "dendrites". The other one, "the axon", is smoothly surfaced and rather straight. It extends downward, bifurcating further below on many points, exhibiting a more regular, somewhat perpendicular pattern. The diameter of the axons may vary from a few microns[4] to hundreds of microns, its length from a few millimeters to a meter or more. Most of these axons terminate on other neurons and establish two different kinds of connections as sketched in Figure 10. One is a direct attachment to the soma of the other cell by formation of an "end bulb", the other one is a somewhat haphazard ascending intertwining with the dendritic ramifications of the target neuron. If one penetrates with a micro-probe the enclosing membrane at any point of the neuron, one finds a change of the electric potential of somewhat less than a tenth of a volt, which indicates that the whole structure in its rest state can be considered as a charged, distributed electric battery.



10. *Schematic (a), and symbolic (b), representation of a single neuron.*

If at the soma this electric potential is momentarily perturbed beyond a certain threshold value, the neuron will "fire", i.e., the perturbation will travel along the axon in form of an electric pulse and will pass on this perturbation to its connectees which, in turn, will respond to this perturbation. You may note the following interesting details in connection with this interaction process. First, the magnitude of the electric pulse is independent of the size of the perturbation. However, a prolonged perturbation in sensory cells will produce a chain of pulses[6] with a repetition frequency approximately proportional to the logarithm of the intensity of the stimulus (Fig. 11). Thus, frequency modulation has long been employed by nature, before it was discovered as a noise-evading method for transmitting signals, and the neat little trick of coding intensity into its logarithm reduces not only the problem of multiplication to simple addition, but also compresses a wide range of intensities into a narrow band of measurable quantities. Second, the magnitude of the traveling pulse does not diminish while it travels along the axon, even if a bifurcation is reached and from then on two pulses travel along their tracks. This is, of course, due to the cleverly distributed battery which supplies the necessary energy at any point along the line. Hence, a set of neurons in series may act as an impressive signal amplifier. Allow me to illustrate this with an example. The dark adapted eye can see in the microscope the flash of a scintillation[5] produced by the decay of a single radioactive atom. This flash corresponds to an energy of about one light quantum. If the observer presses the key of a counter whenever he sees a scintillation, he acts as an amplifier with a gain of about a million billions. A most advanced Electronics Research Laboratory would

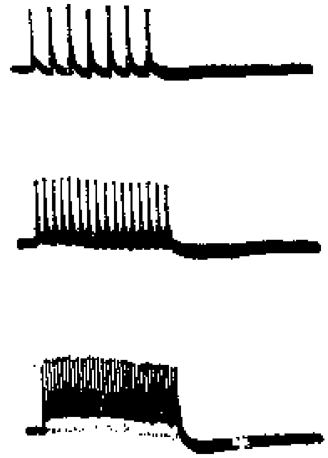
[4]*micron*: one millionth of a meter

[5]*scintillation*: emission of sparks or spark-like flashes of light.

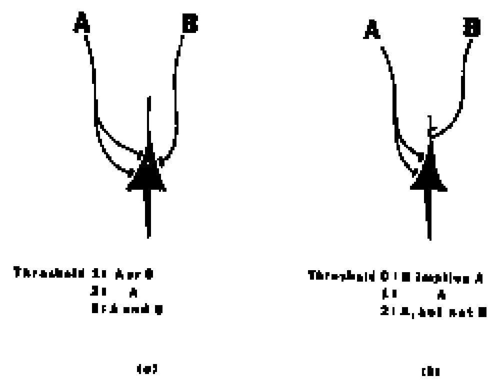
consider the construction of such an amplifier to be on the edge of possibility. But, assuming we had built it, this amplifier would be highly unstable, fluctuating over wide ranges of its output. In the meantime, our observer of scintillations sits quietly on his microscope, eating his sandwich, and does not show the slightest inclination to go into fits if someone sneezes or coughs in the room. He is perfectly stable.

The last point which has to be made with respect to the interaction of neurons is that the two types of connections as sketched in Figure 10a fulfill two different kinds of functions. Connections via end bulbs provide units of excitations, while axons terminating in the dendritic ramifications will inhibit the firing tendency of the target neuron. In other words, if two pulses arrive almost simultaneously at a nerve cell, the one over an end bulb, the other one over the dendrites, their actions will cancel out and nothing will happen. If, however, at a particular instant the only active input is a single end bulb and the threshold of the cell is zero, this will suffice to trip the neuron and it will transmit a pulse via its output axon to other neurons in its connection field.

It was precisely this functional property which suggests the possibility of an idealization of the function of a neuron in form of a logical operation, the affirmative represented by excitation, negatation represented by inhibition. Figure 10b is a symbolic representation of this situation, the triangle standing for the soma, its single upward extension indicating dendritic ramifications with the loop around it symbolizing the inhibitory axon. Excitatory inputs have unmistakably the characteristic end bulbs. Depending upon the symbolic neuron's threshold, which is assumed to change in unit steps, and the arrangement of the input fibers one or more of these idealized elements are capable of computing all possible "logical functions". In Figure 12a, for instance, for a threshold of one unit the neuron will fire when fiber A is active, or when fiber B is active or when both A and B are active, hence the logical function "A *or* B" is computed. Raising the threshold one unit, it will fire always when A is active and logical function "A" is computed; finally if the threshold is three units, only the simultaneous arrival of pulses over A and B will trip the neuron and the function "A *and* B" is computed. Similarly in Figure 12b the functions "B *implies* A", "A" and "A, *but not* B" are computed if the threshold moves through the values 0, 1 and 2. Networks of these idealized neurons can be made capable of computing more and more complicated logical relationships, and, as McCulloch and Pitts[7] showed in their celebrated paper, "A Logical Calculus of the Ideas Immanent in Nervous Activity", any functional behavior which can be defined logically, strictly unambiguously in a finite number of words can also be realized by such a formal neural network.

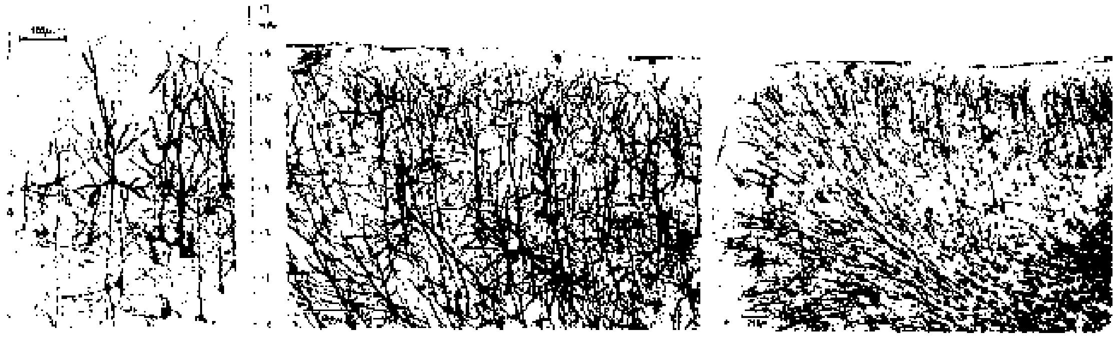


11. *Electrical activity measured with a micro-probe on the axon of a sensory input neuron. (Frequency Modulation).*



12. *Single neurons computing a variety of logical functions.*

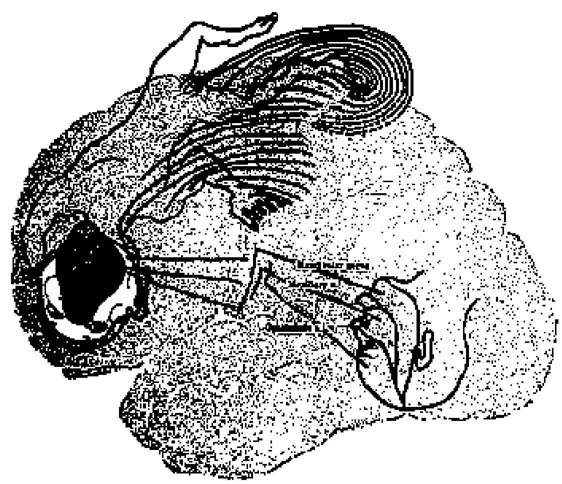
In my opinion this result is perhaps one of the most significant contributions to the theory of knowledge in the last half century. What are the implications of their findings? I believe that they are best summarized in the words of the late John von Neumann,[8] the mathematician. I quote: "It has often been claimed that the activities and functions of the human nervous system are so complicated that no ordinary mechanism could possibly perform them. It has also been attempted to name specific functions which by their nature exhibit this limitation. It has been attempted to show that such specific functions, logically completely described, are *per se* unable of mechanical neural realization. The McCulloch-Pitts result puts an end to this. It proves that anything that can be exhaustively and unambiguously described, anything that can be completely and unambiguously put into words, is *ipso facto* realizable by a suitable finite neural network."



13. *Figures (a)(b)(c) show with decreasing magnification the structure of neural nets in the cortex of a cat.*

Let me show you now a few examples of physiological nerve nets as they can be studied on microscopic slides. Fig. 13a is an enlargement of a small region of the cortex of a cat.[9] The structure that stands out most strongly is a single neuron with its cell-body and a few dendritic ramifications extending upward. It is surrounded by many other neurons with which it may or may not have communicative connections. I may point out that these neurons are made visible by a refined staining method which, however, stains only about 1% of the neurons in this region. You have, therefore, to imagine a dense jungle of such neurons, approximately a hundred times denser as this picture indicates. This accounts for the extraordinary large number of neurons which can be squeezed into a relatively small container as our skull. We carry in our brain approximately ten billion neurons, each of which is a most sophisticated computer element. In order to give an idea of this magnitude I may just mention that presently the earth is inhabited by about three billion people, that is one-third of the number of neurons in our head. Figs. 13b and 13c are magnifications of decreasing power of the same cortical region of the cat, giving you more and more of the whole structural setup. Although you may recognize in 13c a certain directionality and order in the way in which the fibers are aligned, two of the most basic principles of the operation of these nets had to be established by measurements with micro-probes.

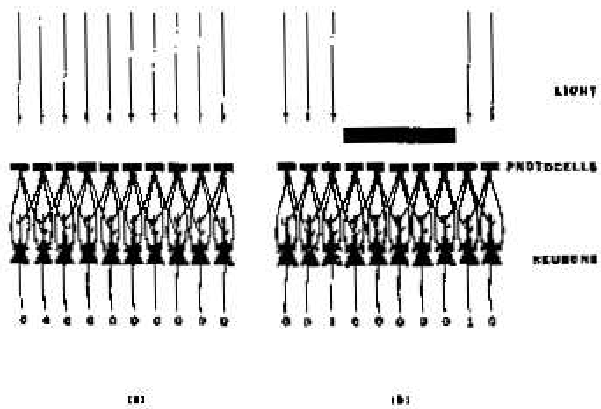
One of these operational principles is a mapping of neighborhoods into neighborhoods, or the principle of "topological mapping". We speak of topological mappings whenever we can set up a continuous one-to-one correspondence between, say, an "object" and its "image", although, by no means, the image has to resemble the object in an obvious way. All geographical maps are, of course, topological mappings of some terrestrial features, all projections including those we have seen earlier and all deformations we may obtain from painting figures on a rubber-balloons and squeezing afterwords this balloon into the craziest shapes. Obviously, in all these transformation procedures neighborhoods will map into neighborhoods and continuity is not destroyed. In many instances our nervous system performs such a topological mapping, particularly when sensory information is projected into the deeper regions of the brain where this information is further processed and reduced. A typical example is the topological preservation of our body with respect to the sensation of touch in the appropriate regions of the brain as you may see in Fig. 14. This "Homunculus" is obtained[10] by registering with micro-probes those regions in the brain which become active when certain regions on the body are stimulated. Thus, we carry in our brain a "signal-representation" of ourselves, with more or less emphasis upon those regions to which evolution allows us to pay more or less attention. Of course, this signal representation is only a preliminary step in further abstractions to which I would like to turn now your attention.



14. *"Homunculus" sensory neighborhoods are mapped into cortical neighborhoods. Topological mapping of the sensation of touch into the outer layers of the brain.*

I am referring to the second basic principle of operation of nerve-nets and in discussing this I have arrived at the core of my presentation. It is only a couple of years ago that Lettvin[11] and a team of neuro-physiologists established this principle beyond doubt in a series of brilliant experiments. Let me briefly describe the experimental set-up. A frog is tied to a small stand, with his head fixed in a certain position. He is surrounded by a white, semi-spherical horizon covering his entire visual field. The illumination of this horizon can be varied, strong shades of objects of various configurations can be projected on this horizon and these shades can be moved about. The frog's response to these optical stimuli is measured in two ways. His muscular action is recorded from the stand to which he is attached. His neuro-optical response is measured by micro-probes which are inserted into single fibers of the optic stalk which is made up of the bundle of nerve-fibers leading from the frog's retina to its "brain". Exposing the frog's visual system to a variety of stimuli, Lettvin and his colleagues found most interesting results. Maintaining the micro-probe in a particular fiber, response was only elicited when a certain kind of stimulus was presented, for instance, when the light was suddenly turned out. This particular fiber remained silent under strongest exposure of light, fast illumination, movement of objects, etc. However, when they moved into another fiber, no response was elicited when the light was suddenly turned out, but strong responses were obtained when, for instance, the shadow of a straight-edge appeared in the visual field. Again this fiber remains inactive for all other kinds of stimuli. In a delightful article entitled "What the Frog's Eye Tells the Frog's Brain", Lettvin and his colleagues reported their findings. Moving from fiber to fiber they found that "the output from the retina of the frog is a set of four distributed operations on the visual image. These operations are independent of the level of general illumination and express the image in terms of: (1) local sharp edges and contrast; (2) the curvature of edge of a dark object: (3) the movement of edges: and (4) the local dimmings produced by movement or rapid general darkening."

At first glance these "operations" appear to be quite mysterious. Are there special fibers which are sensitive only to edges, curves, or other geometric properties? No, this is impossible. Because. first, these properties are distributed properties, revealing themselves only when neighborhoods are inspected; second, all neurons are more or less alike and no specificity for certain stimulation can be expected. It was perhaps an accident that about the same time when these observations were made, we in the Biological Computer Laboratory of the University of Illinois[12] were in the midst of a comprehensive investigation of the computing capabilities of large networks, particularly so called "periodic nets" which are characterized by a periodic repetition of one and the same connection pattern, hence by a repetition of one and the same computational operation. The study of these periodic structures proved to be most rewarding and led to important clues in our understanding of the problem of "cognition" not only in the frog, but also in higher animals, including man.



15. *Periodic net which computes the invariant "Edge", independent of location, strength of illumination and size of object.*

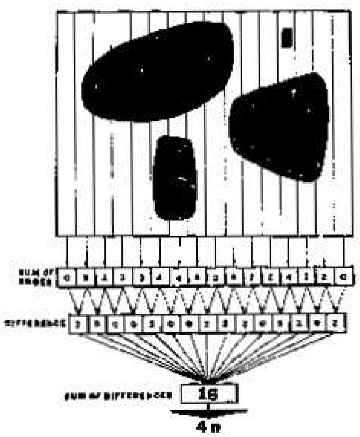
Figure 15a gives an example of such a periodic one-dimensional net. A series of photosensitive elements are connected to a series of corresponding idealized neurons such that the left and right hand neighbor neuron is singly inhibited, while the associated neuron is doubly excited. This connection scheme repeats itself periodically over the entire strip. With all thresholds equal and slightly above zero, clearly, the neural net will not respond, if all photo cells are uniformly illuminated, independent of whether the illumination is strong or faint, because the double inhibition converging on each neuron from photocells to the left and right will cancel the double excitation coming from its corresponding photocell.

An obvious effect of this particular inter-connectivity is its insensitivity to variations of light, despite the fact that the "sensory layer" — the photocells — are all highly sensitive light receptors. You may ask now, why all this effort to make a light sensitive organ insensitive to light? However, you will see in a moment the interesting feature of this net. If now an obstruction is placed into the light path (Fig. 15b), the edge of this obstruction will be detected at once, because the only neuron which will now respond is the one on the edge of the obstruction, receiving insufficient inhibition from only one of its neighbor photocells in the light, while the other one is in the shade and silent. In other words, this net "computes" the invariant "edge", independent of its location and independent of the strength of illumination. The efferent[6] fibers of this network will be active only if edges are present in the visual field of this simple, one-dimensional "retina".

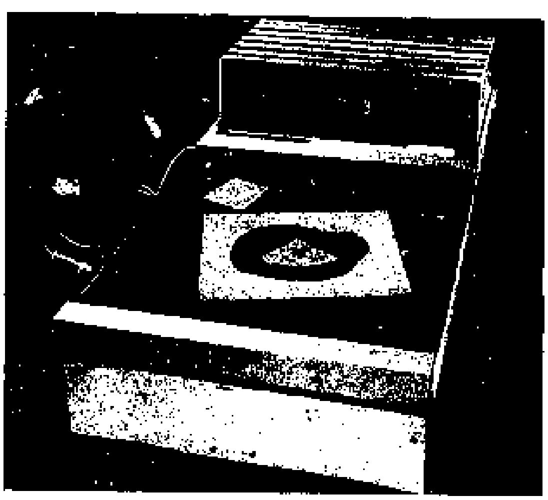
Although the scheme is admittedly simple, this network can detect a property which cannot be detected by the nervous system built into us. Consider the simple topological fact that any finite, one-dimensional obstruction must have two edges (Fig. 15b). If N objects obstruct the light path to our edge detector, 2N neurons will be active and their total output divided by two gives

[6] *efferent*: Conveying outwards, discharging.

exactly the number of objects in the visual field of that "retina". In other words, this strip sees each different number of objects as a different entity, say "seven-ness", "twenty-ness", etc., as we see different electromagnetic frequencies as different colors "red-ness", "green-ness", etc. It might be amusing to see an extension of this principle applied to two dimensions. Fig. 16 sketches a two-dimensional version of an "N-seer" utilizing a series of parallel strips of the previous kind. Each vertical strip reports the number of edges it sees. If the sum of the differences of these values are taken, the result divided by four gives exactly the number of objects seen by this two-dimensional "retina", independent of illumination, size and location of objects, their shape restricted only to the demand that they have to be convex. It may be noted again that no "counting" in the usual sequential sense takes place. The system sees these numbers at one glimpse.



16. *Two-dimensional network computing the invariants "N Convex Objects", independent of their size and location, and independent of strength of illumination.*



17. *The "Numa-Rete", an artificial "retina" counting the number of objects independent of their size, location and form, and independent of strength of illumination.*

The apparent weakness of the simple networks just described, namely to count properly only convex shapes, can be turned into a virtue, if networks with interconnectivities — or, as we say with a specified "neighborhood-logic" — were known which would be able to respond to the number of elements, independent of their shape. Indeed, it is not too difficult to determine the necessary neighborhood-logic and to construct such a device for the purpose of demonstrating this principle of parallel computation. Figure 17 is a picture of our "Numa-Rete" which detects N-ness, independent of illumination, size, location and shape of objects.[13] In spite of the fact that this instrument is not more than an adult toy, its counting capacity is impressive, namely about 20,000 objects per second.

You see now that the deficiency of our previous counter becomes a virtue when combined with the principle of the Numa-Rete, because a parallel combination of these two networks not only sees the number N of objects, but also detects at once the number N_0 of convex objects, a number which must be always equal or less to the number of objects. This difference $(N - N_0)$ may be used to develop the idea "convexity" in order to resolve the inconsistency of the counts of the two networks.

We are today in possession of a general theory of the neighborhood logics which would compute an almost infinite variety of abstracts as, e.g., straightness, curvature, topological connectedness, motion of shapes, flicker etc., in the visual field: chord and timbre independent of pitch, voicing and variation of frequencies typical for definition of spoken phonemes in auditory perception, etc., etc.[14,15] All these abstractors are periodic structures, each period involving from two to at most eight neurons. This may be surprising at first glance. However, it is not difficult to show that the number of different connectivities which can be constructed with n neurons is of the order of

$$2^{n^2}$$

That means that if only five neurons are in the game, 2^{25}, that is more than a billion different nets that can be formed. With more neurons participating in the interaction process, numbers are obtained which dwarf even astronomers in their wildest speculation.[16]

I hope that these few examples made it sufficiently clear that if the interaction scheme or the neighborhood-logic of just a few elements is specified, and this scheme is repeated periodically over an entire layer or elements, this layer will extract a particular property, e.g., an "edge", of the stimulus pattern presented to this layer, independent of other accidental circumstances in this pattern, say, shifts, sizes, stimulus strength, etc. Hence, we may say that such a layer computes an invariant, or carries out an abstraction, or filters out a certain property. Thus, "property-filter" is quite frequently used in reference to such networks.

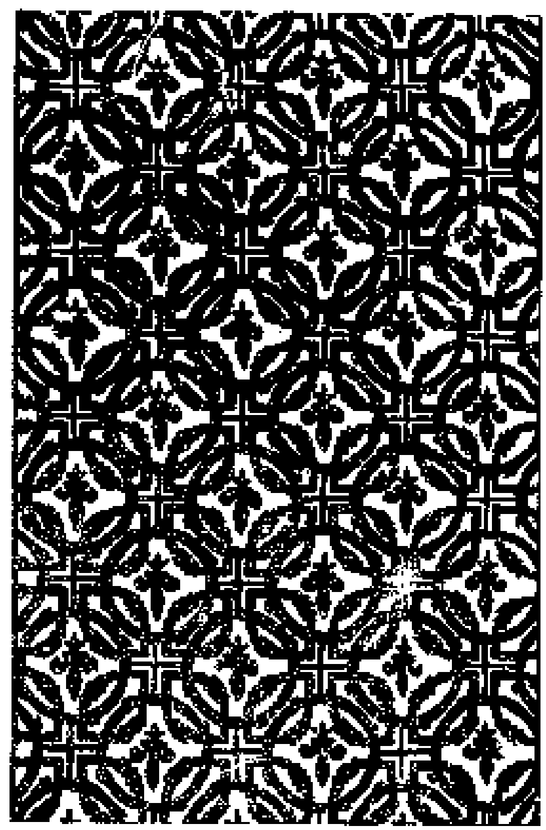
I have first made mention of networks in general and have later introduced the restriction of periodicity. It is significant that this restriction still leaves us with a host of different kinds of property filters, because repetition of a structure is a simple task compared to altering the structure itself. This has of course, been realized by nature again and again. because a genetic command "repeat structure X so and so many times" is a simple and reliable operation, particularly after X has proven itself to be an evolutionary success. However, the command "change structure X into Y" is difficult and risky. Y may turn out to be a flop — on the other hand, it may turn out to be a great success. Only the epigones[7] can tell, and they will always tell a success story, for in the other case there are no epigones!

After we have convinced ourselves that periodicity does not affect the richness of the choice of our structures it is tempting to explore the possibility of a further restriction, namely symmetry of the connection structure or, what amounts to the same, periodicity not only in one or two directions, but also in an angular sense. Our edge-detector of Figure 15 is one example, where the connection structure is not only periodic, but also mirror symmetrical to a symmetry line cutting photo-cell and associated neuron into halves. A similar symmetry is obtained in the two-dimensional system of Figure 16, if the line of symmetry is chosen to be horizontal. A vertical line must not necessarily produce mirror symmetry, because all edge-detector strips may be out of register or may have different periodicity, because there are no connections between strips, hence no "neighborhood" in a horizontal sense. However, connection schemes which are invariant to 90° rotations can be easily imagined and, of course, realized (Numa-Rete). Again, nature has made abundant use of this further structuralization and I am sure you are aware of the many books which display in photographs the innumerable examples of the beautiful regularities, symmetries and periodicities in the fine-structure of living organisms.[17]

The branch of mathematics that investigates the various symmetries and periodicities in spaces of arbitrary dimensions is called "group-theory".[18] A set of operations which transform a geometrical figure into itself is called a group. For instance an equilateral triangle is said to be invariant with respect to the group of rotations of 120°. An arrow is invariant only to the group of rotations of 360°. Hence, the postulate of the invariance of a certain configuration defines a group of operations which, in turn, give sufficient information as to uniquely determine the connection scheme between neighbors in networks of arbitrary dimensionality.

Assume for the moment that a network is formed that is not only periodic, but also extracts at any point certain symmetry-relationships in its stimulus field. I may add that there is considerable neuro-physiological evidence that such nets exist in the visual, but particularly in the auditory system. Imagine now that we expose this network to a stimulus which displays exactly the same symmetry, not only at one point, but all over the stimulus field. Clearly, the whole net will respond at every point; it will, so to say, go into "resonance", and the whole sensory apparatus will be flooded with activity referring to information about this particular pattern. One may compare this situation to the effect on a light-sensitive network which, after being kept for a considerable time in the dark, is suddenly exposed to a fully illuminated environment.

I believe that this explains why early man embellished his artifacts with ornaments and mosaics, and why complex weaving patterns are one of man's earliest achievements in the realization of mathematical concepts. I even venture to say that the almost physical pleasure we experience when exposed to certain highly redundant stimuli is not only at the root of our aesthetic judgment, but is, in fact, intrinsic in the prestructuralization of our nervous system.

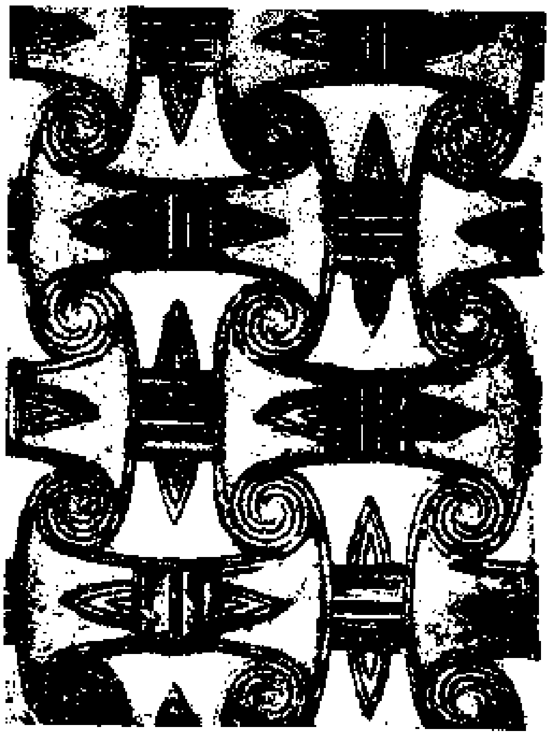


18. *Symmetry Group of the type C_{2v}^{IV}. (Embroidery by Pellegrin, 1530 A.D.).*

It can be shown that in the plane there are exactly 17 different symmetry-groups, two of which are exem-

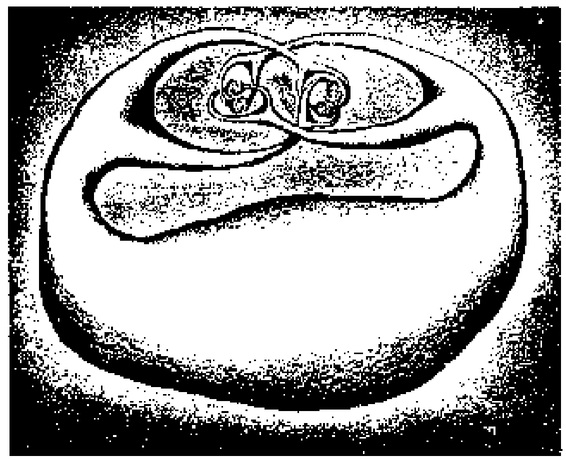
[7] *epigone*: disciple, follower, imitator.

plified in Figures 18 and 19. Fig. 18 is a very simple group with easily detectable symmetries and periodicities.[19] Fig. 19, on the other hand, is a highly sophisticated ornament which belongs to one of the groups with the most complex invariance conditions.[20] Its discovery goes back to about 1600 B.C. which suggests a considerable preoccupation of the early artists with such regularities.



19. *Symmetry Group of the type* $C^{II}_{4\upsilon}$. *(Egyptian ornament, Thebes, about 1600 B.C.).*

If I had to present these points to a group of musicians, I would have a much easier task in pointing out the intrinsic appreciation of certain periodicities and symmetries, because I could easily prove that our whole system of tonality, counterpoint and enharmonic is based on these concepts and the neurophysiology of the corresponding nerve-nets is reasonably well established. In both arts, visual and musical, the creative activity which we usually refer to as "classic", is mainly an exercise in the exploration of acceptable canons, that is structural regularities, that will set into "resonance" some of our intrinsic "configuration detectors". This does not exclude learned appreciation, because what is learnable must present as structural potentiality. We shall never appreciate a painting in ultra-violet, as hard as we may try. As I understand them, some of the modern attempts of artistic creativity aim at a breakthrough to avoid canonicality at any cost. In musical composition this leads to the endeavor of finding the "most improbable" tone in the continuation of a melodic sequence. However, the composer forgets that he and I are chained together by our common genetic make-up, and what is for one human observer the "most improbable" tone is equally improbable for another observer. Hence, having found out that this composition is the sequence "most improbable tones" the listener switches on his "abstractors" tor most improbable tones which become herewith the most probable "most improbable" tones, and are as predictable as my classical sequence, with the only disadvantage that — alas — the physical pleasures I mentioned before are gone. Or, in the world of letters, I would be far more surprised to see the well known signature spelled "E. E. Cummings" instead of "e. e. cummings".



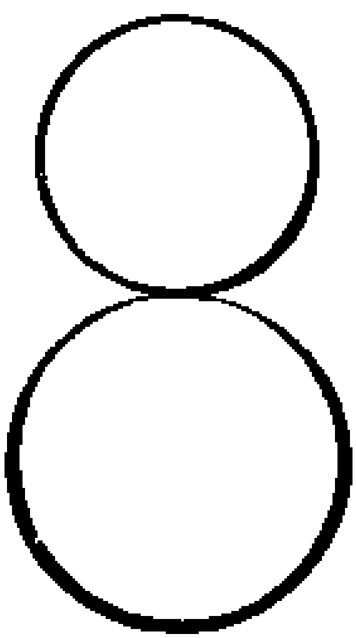
20. *An example of a surface which cannot be topologically mapped into a plane. (Alexander's "Horned Sphere").*

If intellectual bafflement is an artistic goal — and I believe it is a very legitimate goal — then one could make use of a treasure chest filled to the brim with such bafflements, and that is the mathematical branch of topology. I have already referred to topological mappings, whereby neighborhoods are conserved. Clearly, if one knows the conditions under which neighborhoods are conserved, one knows at once also the conditions under which they are not conserved. Since most of our information processing is done under neighborhood conservation conditions, the inverse gives us a kind of intellectual jolt, as for instance a close inspection of Fig. 20 does. Alexander's "Horned Sphere" is a sphere which develops two interlacing horns which develop two interlacing horns which develop two interlacing horns which ... ad infinitum.[21] But aside from the absurdities which are freely offered in topology, this mathematical discipline can be put to use in helping us to explore innumerable relationships in design where we have, I believe, so far only scratched the surface of this wealth.

After this brief excursion into the neurophysiology of aesthetics, permit me to show you briefly two sets of pertinent examples which I think you ought to see before I conclude my presentation.

21. *Correlation between auditory and visual invariants.*

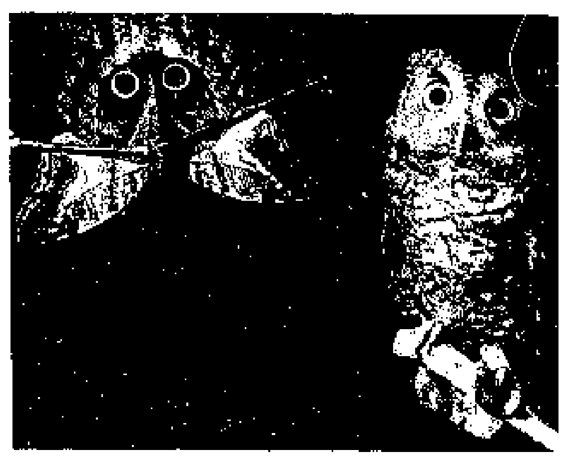
The first set refers to a matching operation with respect to invariants computed by different sensory modalities, or even by different species. The story that goes together with Fig. 21 has it that anthropologists came upon a tribe with unknown language and symbols. The two signs are referred to by the natives as Ooboo and Itratzky. The question arises which one is the Ooboo and which one the Itratzky. The intriguing thing is that everybody knows the answer. How come? Since one of the advantages of multiple sensory modalities is a multiple check on one's environmental structure, it is clear that at higher centers correlations between heterosensory invariants are computed. The example just cited establishes likelihoods between auditory and visual invariants. Fig. 22 gives an example of a correlation between kinesthetic and visual invariants. In its proper presentation the two circles of the figure eight appear to be only slightly different in size. However, turning this figure up side down produces a marked difference in the size of these circles. People with troubles in their semi-circular canals do not register this illusion.



22. *Correlation between kinesthetic and visual invariants.*

The final example in this series is in my opinion most impressive. Fig. 23 shows an owl and a harmless moth embellished with two strong yellow patches mimicking the looks of an owl.[22] Is this one of the peculiar accidents in Nature, or is there something behind this adaptation? The answer is clear if one remembers that one of the tricks in evolution is for an animal to become extremely disgusting and repulsive to its predator, in order to have a better chance of surviving.[23] Birds relish moths, but they also have a fast functioning property detector for "owlishness", which means for the bird "get out of the place as fast as you can". The harmless, defenseless moth has only to spread its wings and gets the desired reaction of its predator. This example suggests to me most convincingly a certain "absoluteness" in form perception.

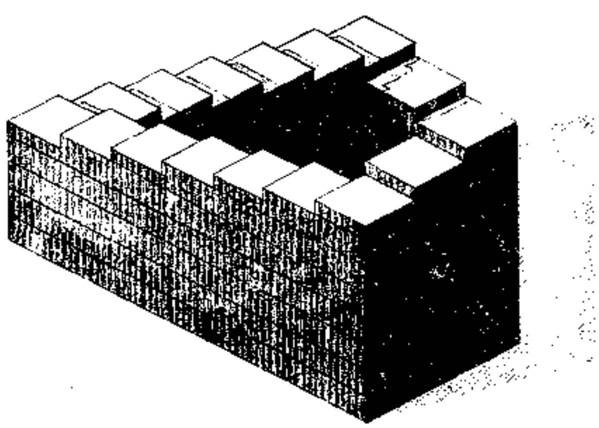
So far I have shown examples where the invariants are computable. However, in many, many instances, this is not the case. And this brings me to my second set of examples. In Fig. 24 you do not see that this engraving of a somewhat peculiar landscape presents "actually" four portraits of the emperors Charles V., Ferdinand I., Paul III, and Francis I. They are presented in form of a point-projection, an "Anamorphosis", a frequently practiced pastime amongst artists in the sixteenth and seventeenth century.[24] Faces of these emperors can easily be recovered by looking at the picture under a flat, grazing angle from the left. Obviously, we do not have built in property-detectors which re-transform this point-projection.



23. *Adaptation to repulsive and distasteful properties.*

24. *An example of man's incapacity to perform projective transformations (Anamorphoses by Erhard Schon, 1535).*

If in the previous example of Fig. 24 you were unable to see the things which were there, in my last example (Fig. 25), you see something which is not only not there, it does not even exist! It is an impossible object.[25] You convince yourself of the impossibility of this object by attempting to go down the circular staircase. You go down, down, down ... but you are, still there, where you started. While in the case of anamorphoses we fail to compute the appropriate invariants, in the case of the deceptive staircase our invariants are computed a little bit too hastily. It is comforting to know that our natural environment does not usually play such dirty tricks with us — if it did, the last two billion years of evolution would have taught us to take them in stride.



25. *An impossible object.*

I have now arrived at the conclusion of my presentation and I am somewhat afraid that I may have given the impression that our brain is stuffed with sophisticated computers extracting invariants after invariants. May I first dispel this impression by showing that this would not even be necessary. Consider a system that can compute p independent invariants, in other words, a system which has p property filters incorporated. For a given stimulus pattern some of these filters will respond, some will not, depending upon whether or not the corresponding properties are present in the stimulus. We may ask, how many different combinations of on-and-off states of the p filters in our system are possible? The answer to this question is interesting because it gives us the number of different environments our system is capable to distinguish and since each of these distinguishable environments may elicit a different response — or a different "word" — we may get an idea of the richness of its vocabulary. The answer to our question is very simple indeed, the number of distinguishable states, or words, is

$$W = 2p$$

Applied to the frog with his four filters, we have for $p = 4$:

$$W = 2^4 = 16$$

That is, a frog has a vocabulary of 16 words, each of which has exceptional survival values for him. We have about 200,000 words in our vocabulary. Only eighteen such filters ($p = 18$) would be sufficient to produce this rich variety in our appreciation of the world we live in. This result is as absurd as the other one where we were led to believe that the number of these computers may go into the thousands or even into the ten-thousands. The resolution of this discrepancy is given by an important function of the brain, namely its adaptability and flexibility, its dynamic capacity, to do and to develop quickly many of the required operations.

I confined myself today to present to you the Twentieth Century version[26] of a classical concept, namely the concept of a "Platonic Idea". These ideas refer to the invariants which comprise our knowledge before experience and learning. If we would recognize and cultivate in our youngsters these intrinsic ordering principles, I am convinced, with Professor Chermayeff, that top men will leave our Institutions many years younger than they leave today.[27]

Notes

[1]Hilbert, D. and Cohn-Vossen, S.: "Anschauliche Geometrie," Gr. Math. Wissi XXXVII, Springer, Berlin 1936, and Dover, New York, 1944, p. 133.

[2]Ritchie, A. C.: "Sculpture of the Twentieth Century," Mus. Mod. Art, New York, 1952, p. 89.

[3]Ref 2, p. 77.

[4]Brandt, B.: "Perspectives sur le Nu," Editions Prisma, Paris, 1961, p. 12.

[5]Scholl, D. A.: "The Organization of the Cerebral Cortex," Methuen and Cie., London, 1956, p. 3.

[6]Eccles, J. C.: "The Neurophysical Basis of Mind," The Clarendon Press, Oxford, 1952, p. 122.

[7]McCulloch, W. S. and Pitts, W.: "A Logical Calculus of the Ideas Immanent in Nervous Activity," Bull. Math. Biophys. 5, 115–133 (1943).

[8] von Neumann, J.: ”The General and Logical Theory of Automata,” *Cerebral Mechanisms of Behavior*, Jeffress, L. A. (ed.), John Wiley & Sons, New York, 1951, p. 21 ff.

[9] Ref 5. plate 6, 12 and 3.

[10] Mettler, F. A.: “Neuroanatomy,” The C. V. Mosby Co., St. Louis, 1948,p. 432.

[11] Lettvin, J. Y., Maturana, H. R., McCulloch, W. S., Pitts, W.: ”What the Frog’s Eye tells the Frog’s Brain,” Proc. IRE 47, 1940–1951 (1959).

[12] Babcock, M. L., Inselberg, A., Lofgren, L., Weston, P. and Zopf, G. W., Jr.: “Some Principles of Pre-Organization in Self-Organizing Systems,” Contract Nonr 1834(21) Tech. Rep. #2; Electrical Engineering Laboratory, Engineering Experiment Station, University of Illinois, 1960.

[13] Weston, P.: ”Photocell Field Counts Random Objects,” Electronica, pp. 44–46, September 22, 1961.

[14] Babcock, M. L.; Erickson, R. J. and Neill, D. M.: ”A Dynamic Signal Analyzer,” Contract AF 6428, Tech. Rep. No. 3–1, Electrical Engineering Research Laboratory, Engineering Experiment Station, University of Illinois, Urbana, Illinois, 1962.

[15] Inselberg, A. and Von Foerster, H.: ”Property Extraction in Linear Networks,” NSF Grant 17414, Tech. Rep. No. 2, Electrical Engineering Research Laboratory, Engineering Experiment Station, University of Illinois, Urbana, Illinois, 1962.

[16] Von Foerster, H.: ”Some Remarks on the Design of Biological Computers,” 2. Intern. Congr. Cyb., Assoc. Int. de Cybernetique, Namur, 1960, pp. 241–255.

[17] Postma, C.: ”Plant Marvels in Miniature,” The John Day Co., Inc. 1961.

[18] Weyl, H.: ”Symmetry,” Princeton U.P., Princeton, 1952.

[19] Lehner, E.: ”Alphabets and Ornaments,” The World Publ. Co., New York, 1952, p. 159.

[20] Ref 18, p. 113.

[21] Hocking, J. G. and Young, G. S.: ”Topology,” Addison-Wesley Pub. Co. Inc., London, 1961, p. 176.

[22] Life Magazine.

[23] Fisher, Sir Ronald, A.: “The Genetical Theory of Natural Selection,” Dover Publ. Inc., New York, 1958, p. 177.

[24] Baltrusatis, J.: “Anamorphoses,” Olivier Perrin Editeur, Paris, 1955, p. 14.

[25] Penrose, L. S. and Penrose, R.: ”Impossible Objects; A Special Type of Visual Illusion,” Br. Jour. Psy.: vol. 49, 31–33 (1958).

[26] Von Foerster, H. ”Circuitry of Clues to Platonic Ideation,” Biosimulation, Muses, C. (ed) Falcon Press, 1962.

[27] The work reported herein is sponsored in part by the National Science Foundation under NSF Grant 17414.