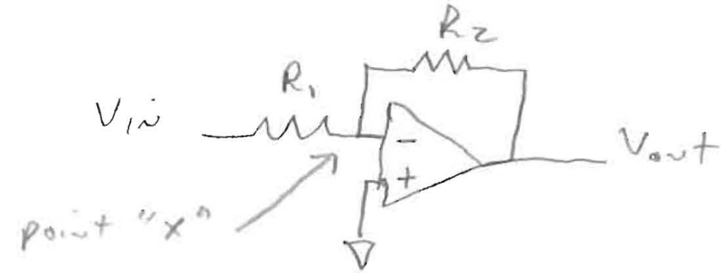
Circuits Homework Solutions
Week 6, Motion 2015

#1)



"Inverting amplifier"

Since the "+" ("non-inverting") input is grounded, the voltage at the "−" input must also be zero (due to Golden Rule #1, the output will do whatever it can to make the difference between the "+" and "−" inputs become zero)

In that case, the voltage accross R_1 is $\Delta V_1 = V_{in} - 0 = V_{in}$, so the current through R_1 is

$$I_1 = \frac{\Delta V_1}{R_1} = \frac{V_{in}}{R_1} \qquad (1)$$

The op-amp inputs draw no current (Golden Rule #2), so the current I_1 must continue on through R_2. But $\Delta V_2 = 0 - V_{out} = -V_{out}$ (2) since we already know that the voltage at the "−" input is zero (because we grounded the "+" input).

So $I_1 R_2 = \Delta V_2 = -V_{out}$ (3)

or $V_{out} = -I_1 R_2 = -\frac{R_2}{R_1} V_{in}$

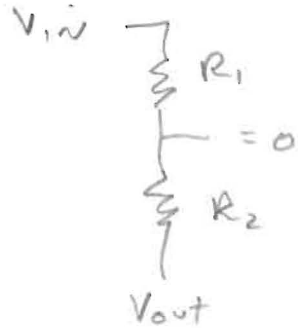
Giving $\boxed{G = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$ (4)

note: $G = -10$ for $R_1 = 1K$ and $R_2 = 10K$

#1 cont'd

Note that you could also do this by the following (more complicated, but correct) strategy:

Since the "−" input must be at 0 volts,



= 0 V at this point, which is point X in the last Figure.

But V_x, the voltage at point X, is equal to

$$V_x = V_{out} + (V_{in} - V_{out}) \cdot \frac{R_2}{R_1 + R_2} \qquad (5)$$

using the voltage divider equation and given that the bottom of the voltage divider is at V_{out} instead of 0 V (ground) the way we usually see it.

Since $V_x = 0$, we can rearrange Eqn 5 to get

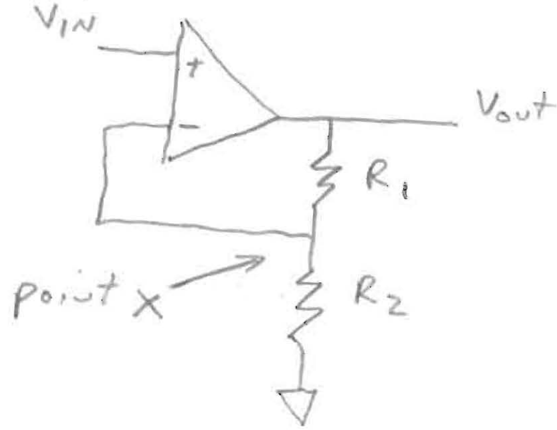
$$V_{out}\left(1 - \frac{R_2}{R_1+R_2}\right) = -V_{in}\left(\frac{R_2}{R_1+R_2}\right) \qquad (6)$$

or $$V_{out}\left(\frac{R_1}{R_1+R_2}\right) = -V_{in}\left(\frac{R_2}{R_1+R_2}\right) \qquad (7)$$

so $$\boxed{\frac{V_{out}}{V_{in}} = G = -\frac{R_2}{R_1}} \qquad (8)$$, as before.

Either way of solving it is fine.

#2



"non-inverting amplifier"

Well, V_x, the voltage at point x, must be

$$V_x = \frac{R_2}{R_1+R_2} V_{out} \quad \text{per the voltage divider equation} \quad (1)$$

and

$$V_x = V_{in} \quad \text{per Golden Rule \#1 (output adjusts to make the inputs equal)} \quad (2)$$

So

$$V_{in} = V_x = V_{out} \cdot \frac{R_2}{R_1+R_2} \quad (3)$$

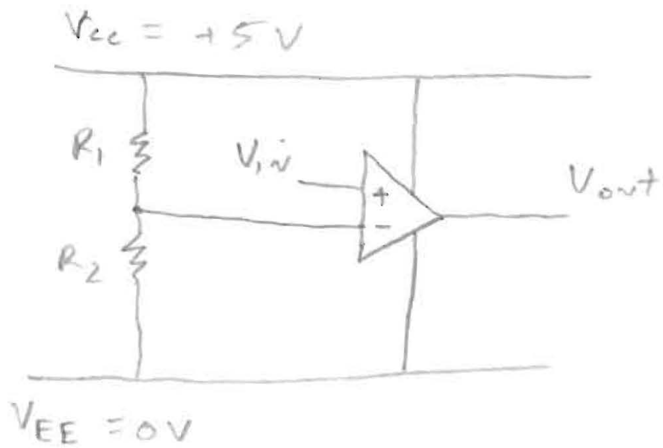
or

$$\boxed{G = \frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2}} \quad (4)$$

where $G = 11$ for $R_1 = 1k$ and $R_2 = 10k$.

#3) Given that the op-amp is powered from $V_{cc} = +5V$ and $V_{EE} = 0V$ (V_{cc} = positive supply voltage "rail", and V_{EE} = negative supply voltage rail – these are standard names in circuit design) we know the op-amp output ~~cannot~~ go above +5V or below 0V... for now, assume the op-amp can get to these values (this is known as "rail-to-rail output"; not all op-amps can do that.)

Then if we have



where we pick $R_1 = R_2$ so that the voltage at the "−" input is 2.5V,

→ when $V_{in} > 2.5V$ ⇒ output goes as positive (1)

as possible, since $V_{out} = A \cdot (V_+ - V_-)$, where A is a large, positive number (called the "open-loop gain", ie. the gain when no feedback – a connection of some sort between the output and the input(s) – is included.)

3 cont'd

so if $V_+ > 2.5\,V$, the output will go to

$$V_{out} = A\,(V_+ - 2.5\,V) = \infty \quad \text{(roughly; } A \text{ is usually very big)} \qquad (1)$$

But since the positive supply is only 5 V,

$$V_{out} \to +5\,V \quad \text{when } V_{in} = V_+ > 2.5\,V \qquad (2)$$

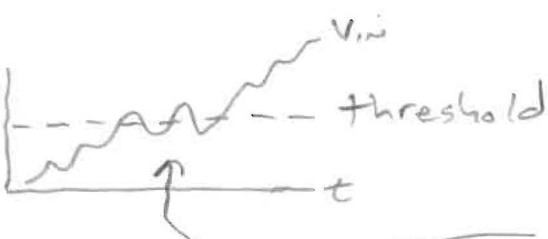
Similarly, if $V_+ < 2.5\,V = V_-$, $V_{out} \to -\infty$ (3)

or, since the negative supply is 0V,

$$V_{out} \to 0\,V \quad \text{when } V_{in} = V_+ < 2.5\,V \qquad (4)$$

Note that by choosing R_1 and R_2, you can adjust the "threshold voltage" at which the transition occurs.

// Further note: if your input is noisy, the output might switch between +5 V and 0V several times as V_{in} passes the threshold



output switches every time noisy signal passes the threshold.

The solution to this is to use the output to shift the threshold a little bit (more than the noise level). That circuit is called a "Schmidt trigger".

#4)

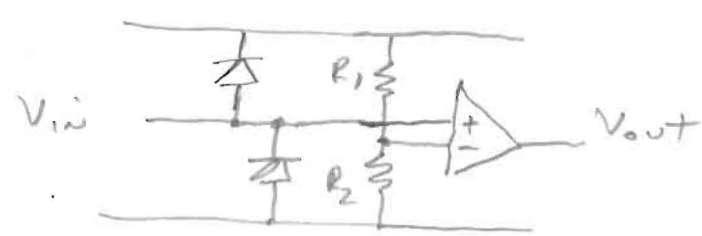
To prevent the input V_{in} from going above the positive supply rail, we can use a diode:

With the diode, if $V_{in} > V_{supply}$, current will flow through the diode preventing V_{in} from rising further (until the diode burns out).

Of course, there will be a $\simeq 0.6$ V diode drop, so

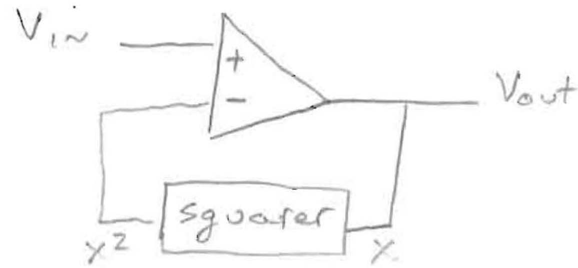
$$V_{in} \leq 0.6\,V + V_{supply} = 5.6\,V \qquad (1)$$

We can add a similar protection diode to the negative supply so that $V_{in} \geq -0.6\,V$ (2)



Diodes used like this are called "protection diodes" or diode "limiters."

#5)



From Golden Rule #1 we know $V_+ = V_-$ (1)

and the "squarer" in the feedback loop requires

$$V_- = V_{out}^2 \qquad (2)$$

So we have

$$V_{in} = V_+ = V_- = V_{out}^2 \qquad (3)$$

or

$$\boxed{V_{out} = \sqrt{V_{in}}} \qquad (4)$$

Notice that the circuit produces the <u>inverse</u> of what you put in the feedback loop... that is a general thing.

You might ask yourself, "yes, but will the output be the positive or the negative square root?" After all, Eqn (3) is ambiguous re: the sign of Vout. The easy way to decide about this is to consider what happens if Vin changes.

#5 cont'd

Say $V_{in} = 4V$, and $V_{out} = -2V$ (consistent w/ Eqn (3)), and both V_+ and $V_- = 4V$.

Now let $V_{in} = V_+$ rise to $4.1\ V$ ... since

$V_{out} = A(V_+ - V_-)$, with A = large, positive (5)

and $V_+ = 4.1\ V$ and $V_- = 4V$, $V_{out} = A \cdot (0.1V) \geq 0$ (6)

so V_{out} will go <u>up</u>.

But $V_{out} = -2V$, so when it goes <u>up</u>, V_{out}^2 will go <u>down</u>.

which will make $(V_+ - V_-)$ even bigger, so V_{out} will go <u>up</u> even more ... until $V_{out} = +\sqrt[2]{4.1}\ V$ ...

On the other hand, if $V_{in} = 4V$ and $V_{out} = +2V$, and $V_{in} = V_+$ goes up to $4.1\ V$, the same arguments as above will result in V_{out} going <u>up</u> ... and it will get (very) close to $V_{out} = +\sqrt[2]{V_{in}}$ and settle there.

$\Rightarrow$ So the circuit produces the positive square root.

N.b. if it now occurs to you that $V_{out} = 0$ unless $(V_+ - V_-) \neq 0$, so Golden Rule #1 can't be completely correct, you are right. But it is usually <u>very</u> close to correct, as we'll see later.