Models of Motion, Week 2 Solutions | A. E, chapter 1

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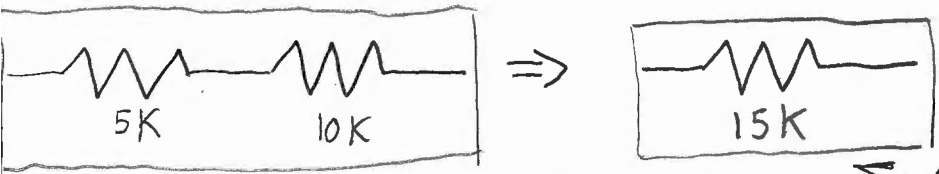
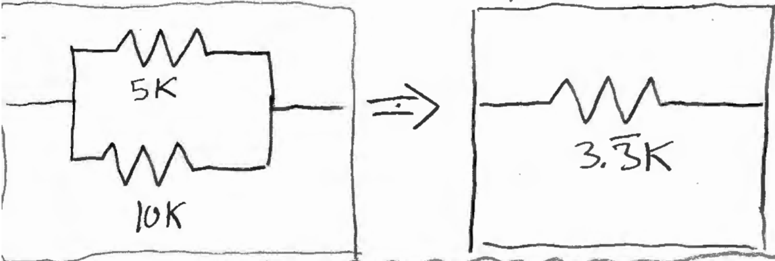
4/10/15

1. For resistors in series $R_{Total} = \sum_n R_n$



For resistors in parallel $1/R_{Total} = \sum_n 1/R_n$
For two resistors in parallel $R_{Total} = R_1 R_2/(R_1 + R_2)$



2. $R = 1\Omega$ $\quad V = 12V \quad P = IV = V^2/R = \boxed{144\,W}$

3. $V_{Total} = \sum_n V_n \Rightarrow I R_{Total} = \sum_n I R_n = I \sum_n R_n$ (Current is the same through resistors in series)

$$\boxed{R_{Total} = \sum_n R_n}$$

For resistors in parallel current can vary, but voltage is the same, the sum of currents through The current through all of the resistors is the sum of currents through all the resistors,

$V_{in} = V_n \qquad I_{Total} = \sum_n I_n \qquad V = IR \Rightarrow I = V/R$

$$\frac{V_{in}}{R_T} = \sum_n \frac{V_{in}}{R_n} = V_{in} \sum_n \frac{1}{R_n}$$

$$\boxed{\frac{1}{R_T} = \sum_n \frac{1}{R_n}}$$

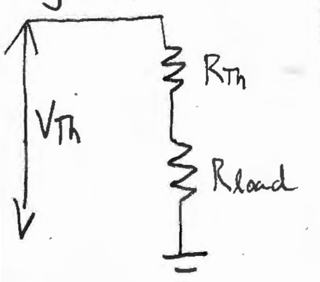
$\boxed{5}$ Power rating $= \frac{1}{4}W = 0.25W$, $R > 1K$ $\quad V = 15V$

$P = \frac{V^2}{R} = \frac{225\ Volts^2}{R}$ For $R = 1K = 1000\Omega$ $\quad P = \frac{225}{1000} W = 0.225W$

$\boxed{0.225W < 0.25W}$, increasing R only decreases P for given voltage, and connecting it in various ways can not increase voltage.

$\boxed{10}$ Show $R_{load} = R_{source}$ maximizes power in load for given source resistance $\quad R_{source} = \text{constant}$

Fig 1.10



Just before exercise 1.10 under "Power Transfer" it gives $R_{source} = R_{internal} = R_{Th}$

Want to maximize $P_l = I_l V_l$

Since $(R_{Th} = R_s)$ and R_l are in series $I_s = I_l = \frac{V_{Th}}{R_s + R_l}$

so $P_l = \frac{V_{Th}}{R_s + R_l} V_l = \left(\frac{V_{Th}}{R_s + R_l}\right)\left(I_l R_l\right) = \left(\frac{V_{Th}}{R_s + R_l}\right)^2 R_l$

V_{Th} is constant and $R_s = R_{source} = \text{constant}$

To maximize P_l with respect to R_l, differentiate with respect to R_l and set the derivative $= 0$

(Quotient rule)

$P_l = V_{Th}^2 \frac{R_l}{(R_s + R_l)^2}$ $\qquad \frac{\partial P_l}{\partial R_l} = V_{Th}^2 \left[\frac{(R_s + R_l)^2 - R_l \cdot 2(R_s + R_l)}{(R_s + R_l)^2}\right] = 0$

$$(R_s + R_l)^2 - R_l \cdot 2(R_s + R_l) = 0$$

$$R_s^2 + \underline{2R_s R_l} + R_l^2 - \underline{2R_l R_s} - 2R_l^2 = 0$$

$$R_s^2 - R_l^2 = 0$$

$$R_s^2 = R_l^2$$

$$\boxed{R_s = R_l} \quad (\text{assuming } R_l \geq 0;\ R_s \geq 0)$$

To show it is maximum take $\frac{\partial^2 P_l}{\partial R_l^2}$ and make sure it is less than zero, but that is probably unecessary.

$\boxed{1.17}$ Show if $A = BC$
then $|A| = |B||C|$ where A, B, C are complex numbers

$$\boxed{\begin{aligned} A = a+ib &= |A|e^{i\theta} \\ B &= |B|e^{i\phi} \\ C &= |C|e^{i\gamma} \end{aligned}}$$

Where θ, ϕ, γ are arbitrary

$$A = BC$$

$$\boxed{|A|e^{i\theta}} = |B|e^{i\phi}\,|C|e^{i\gamma} = \boxed{|B||C|e^{i(\phi+\gamma)}}$$

since $A = BC$, $|A|^2 = |BC|^2$
so taking the magnitude of both sides

$$\left| |A|e^{i\theta} \right|^2 = \left| |B||C|e^{i(\phi+\gamma)} \right|^2$$

$$= |A|e^{i\theta}\left(|A|e^{-i\theta}\right) = |B||C|e^{i(\phi+\gamma)}\left(|B||C|e^{-i(\phi+\gamma)}\right)$$

$$|A|^2 e^{i(\theta-\theta)} = |B|^2|C|^2 e^{i(\phi+\gamma-\phi-\gamma)}$$

$$|A|^2 e^0 = (|B||C|)^2 e^0$$

$$\sqrt{|A|^2} = \sqrt{(|B||C|)^2}$$

$$|A| = |B||C|$$ since magnitudes are real and positive by definition

Q.E.D.