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CONCEPTS

QUANTITATIVE TOOLS

Chances are you are taking this course in physics because someone told you to take it, and it may not be clear to you *why* you should be taking it. One good reason for taking a physics course is that, first and foremost, physics provides a fundamental understanding of the world. Furthermore, whether you are majoring in psychology, engineering, biology, physics, or something else, this course offers you an opportunity to sharpen your reasoning skills. Knowing physics means becoming a better problem solver (and I mean *real* problems, not textbook problems that have already been solved), and becoming a better problem solver is empowering: It allows you to step into unknown territory with more confidence. Before we embark on this exciting journey, let's map out the territory we are going to explore so that you know where we are going.

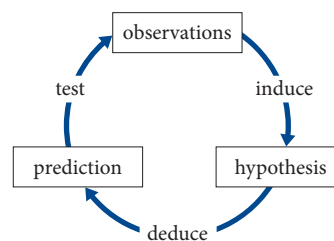
1.1 The scientific method

Physics, from the Greek word for “nature,” is commonly defined as the study of matter and motion. Physics is about discovering the wonderfully simple unifying patterns that underlie absolutely everything that happens around us, from the scale of subatomic particles, to the microscopic world of DNA molecules and cells, to the cosmic scale of stars, galaxies, and planets. Physics deals with atoms and molecules; gases, solids, and liquids; everyday objects, and black holes. Physics explores motion, light, and sound; the creation and annihilation of matter; evaporation and melting; electricity and magnetism. Physics is all around you: in the Sun that provides your daylight, in the structure of your bones, in your computer, in the motion of a ball you throw. In a sense, then, physics is the study of all there is in the universe. Indeed, biology, engineering, chemistry, astronomy, geology, and so many other disciplines you might name all use the principles of physics.

The many remarkable scientific accomplishments of ancient civilizations that survive to this day testify to the fact that curiosity about the world is part of human nature. Physics evolved from *natural philosophy*—a body of knowledge accumulated in ancient times in an attempt to explain the behavior of the universe through philosophical speculation—and became a distinct discipline during the scientific revolution that began in the 16th century. One of the main changes that occurred in that century was the development of the **scientific method**, an iterative process for going from observations to validated theories.

In its simplest form, the scientific method works as follows (**Figure 1.1**): A researcher makes a number of observations concerning either something happening in the natural world (a volcano erupting, for instance) or something happening during a laboratory experiment (a dropped brick and a dropped Styrofoam peanut travel to the floor at different speeds). These observations then lead the researcher to formulate a **hypothesis**, which is a tentative explanation of the observed phenomenon. The hypothesis is used to predict

Figure 1.1 The scientific method is an iterative process in which a hypothesis, which is inferred from observations, is used to make a prediction, which is then tested by making new observations.



the outcome of some related natural occurrence (how a similarly shaped mountain near the erupting volcano will behave) or related laboratory experiment (what happens when a book and a sheet of paper are dropped at the same time). If the predictions prove inaccurate, the hypothesis must be modified. If the predictions prove accurate in test after test, the hypothesis is elevated to the status of either a **law** or a **theory**.

A law tells us *what* happens under certain circumstances. Laws are usually expressed in the form of relationships between observable quantities. A theory tells us *why* something happens and explains phenomena in terms of more basic processes and relationships. A scientific theory is not a mere conjecture or speculation. It is a thoroughly tested explanation of a natural phenomenon, one that is capable of making predictions that can be verified by experiment. The constant testing and retesting are what make the scientific method such a powerful tool for investigating the universe: The results obtained must be repeatable and verifiable by others.

Exercise 1.1 Hypothesis or not?

Which of the following statements are hypotheses? (a) Heavier objects fall to Earth faster than lighter ones. (b) The planet Mars is inhabited by invisible beings that are able to elude any type of observation. (c) Distant planets harbor forms of life. (d) Handling toads causes warts.

SOLUTION (a), (c), and (d). A hypothesis must be experimentally verifiable. (a) I can verify this statement by dropping a heavy object and a lighter one at the same instant and observing which one hits the ground first. (b) This statement asserts that the beings on Mars cannot be observed, which precludes any experimental verification and means this statement is not a valid hypothesis. (c) Although we humans currently have no means of exploring or closely observing distant planets, the statement is in principle testable. (d) Even though we know this statement is false, it is verifiable and therefore is a hypothesis.

Because of the constant reevaluation demanded by the scientific method, science is not a stale collection of facts but rather a living and changing body of knowledge. More important, any theory or law *always* remains tentative, and the testing never ends. In other words, it is not possible to

prove any scientific theory or law to be absolutely true (or even absolutely false). Thus the material you will learn in this book does not represent some “ultimate truth”—it is true only to the extent that it has not been proved wrong.

A case in point is *classical mechanics*, a theory developed in the 17th century to describe the motion of everyday objects (and the subject of most of this book). Although this theory produces accurate results for most everyday phenomena, from balls thrown in the air to satellites orbiting Earth, observations made during the last hundred years have revealed that under certain circumstances, significant deviations from this theory occur. It is now clear that classical mechanics is applicable for only a limited (albeit important) range of phenomena, and new branches of physics—*quantum mechanics* and the theory of *special relativity* among them—are needed to describe the phenomena that fall outside the range of classical mechanics.

The formulation of a hypothesis almost always involves developing a **model**, which is a simplified conceptual representation of some phenomenon. You don’t have to be trained as a scientist to develop models. Everyone develops mental models of how people behave, how events unfold, and how things work. Without such models, we would not be able to understand our experiences, decide what actions to take, or handle unexpected experiences. Examples of models we use in everyday life are that door handles and door hinges are on opposite sides of doors and that the + button on a TV remote increases the volume or the channel number. In everyday life, we base our models on whatever knowledge we have, real or imagined, complete or incomplete. In science we must build models based on careful observation and determine ways to fill in any missing information.

Let’s look at the iterative process of developing models and hypotheses in physics, with an eye toward determining what skills are needed and what pitfalls are to be avoided (Figure 1.2). Developing a scientific hypothesis often begins

with recognizing patterns in a series of observations. Sometimes these observations are direct, but sometimes we must settle for indirect observations. (We cannot directly observe the nucleus of an atom, for instance, but a physicist can describe the structure of the nucleus and its behavior with great certainty and accuracy.) As Figure 1.2 indicates, the patterns that emerge from our observations must often be combined with simplifying assumptions to build a model. The combination of model and assumptions is what constitutes a hypothesis.

It may seem like a shaky proposition to build a hypothesis on assumptions that are accepted without proof, but making these assumptions—*consciously*—is a crucial step in making sense of the universe. All that is required is that, when formulating a hypothesis, we must be aware of these assumptions and be ready to revise or drop them if the predictions of our hypothesis are not validated. We should, in particular, watch out for what are called *hidden assumptions*—assumptions we make without being aware of them. As an example, try answering the following question. (Turn to the final section of the *Principles* volume, “Solutions to checkpoints,” for the answer.)



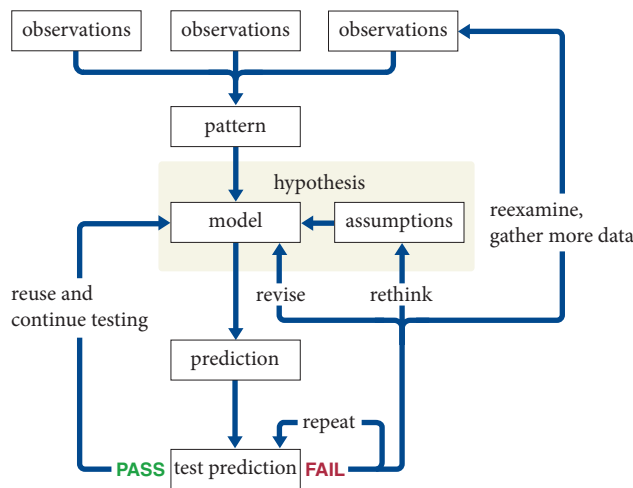
1.1 I have two coins in my pocket, together worth 30 cents. If one of them is not a nickel, what coins are they?

Advertising agencies and magicians are masters at making us fall into the trap of hidden assumptions. Imagine a radio commercial for a new drug in which someone says, “Baroxan lowered my blood pressure tremendously.” If you think that sounds good, you have made a number of assumptions without being aware of them—in other words, hidden assumptions. Who says, for instance, that lowering blood pressure “tremendously” is a good thing (dead people have tremendously low blood pressure) or that the speaker’s blood pressure was too high to begin with?

Magic, too, involves hidden assumptions. The trick in some magic acts is to make you assume that something happens, often by planting a false assumption in your mind. A magician might ask, “How did I move the ball from here to there?” while in reality he is using two balls. I won’t knowingly put false assumptions into *your* mind in this book, but on occasion you and I (or you and your instructor) may unknowingly make different assumptions during a given discussion, a situation that unavoidably leads to confusion and misunderstanding. Therefore it is important that we carefully analyze our thinking and watch for the assumptions that we build into our models.

If the prediction of a hypothesis fails to agree with observations made to test the hypothesis, there are several ways to address the discrepancy. One way is to rerun the test to see if it is reproducible. If the test keeps producing the same result, it becomes necessary to revise the hypothesis, rethink the assumptions that went into it, or reexamine the original observations that led to the hypothesis.

Figure 1.2 Iterative process for developing a scientific hypothesis.



Exercise 1.2 Dead music player

A battery-operated portable music player fails to play when it is turned on. Develop a hypothesis explaining why it fails to play, and then make a prediction that permits you to test your hypothesis. Describe two possible outcomes of the test and what you conclude from the outcomes. (*Think before you peek at the answer below.*)

SOLUTION There are many reasons the player might not turn on. Here is one example. Hypothesis: The batteries are dead. Prediction: If I replace the batteries with new ones, the player should work. Possible outcomes: (1) The player works once the new batteries are installed, which means the hypothesis is supported; (2) the player doesn't work after the new batteries are installed, which means the hypothesis is not supported and must be either modified or discarded.



1.2 In Exercise 1.2, each of the conclusions drawn from the two possible outcomes contains a hidden assumption. What are the hidden assumptions?

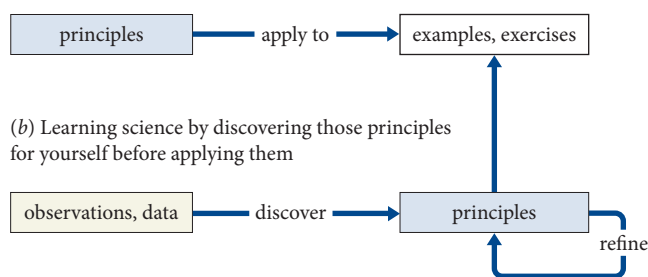
The development of a scientific hypothesis is often more complicated than suggested by Figures 1.1 and 1.2. Hypotheses do not always start with observations; some are developed from incomplete information, vague ideas, assumptions, or even complete guesses. The refining process also has its limits. Each refinement adds complexity, and at some point the complexity outweighs the benefit of the increased accuracy. Because we like to think that the universe has an underlying simplicity, it might be better to scrap the hypothesis and start anew.

Figure 1.2 gives an idea of the skills that are useful in doing science: interpreting observations, recognizing patterns, making and recognizing assumptions, thinking logically, developing models, and using models to make predictions. It should not come as any surprise to you that many of these skills are useful in just about any context. Learning physics allows you to sharpen these skills in a very rigorous way. So, whether you become a financial analyst, a doctor, an engineer, or a research scientist (to name just a few possibilities), there is a good reason to take physics.

Figure 1.1 also shows that doing science—and physics in particular—involves two types of reasoning: *inductive*, which is arguing from the specific to the general, and *deductive*, arguing from the general to the specific. The most creative part of doing physics involves inductive reasoning, and this fact sheds light on how you might want to learn physics. One way, which is neither very useful nor very satisfying, is for me to simply tell you all the general principles physicists presently agree on and then for you to apply those principles in examples and exercises (Figure 1.3a). This approach involves deductive reasoning only and robs you of the opportunity to learn the skill that is the most likely to benefit your career: discovering underlying patterns. Another way is for me to present you with data and observations and make you part of the discovery and refinement of the physics principles (Figure 1.3b). This approach

Figure 1.3

(a) Learning science by applying established principles



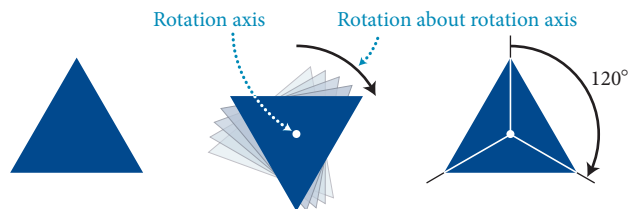
is more time-consuming, and sometimes you may wonder why I'm not just *telling* you the final outcome. The reason is that discovery and refinement are at the heart of doing physics!



1.3 After reading this section, reflect on your goals for this course. Write down what you would like to accomplish and why you would like to accomplish this. Once you have done that, turn to the final section of the *Principles* volume, "Solutions to checkpoints," and compare what you have written with what I wrote.

1.2 Symmetry

One of the basic requirements of any law of the universe involves what physicists call *symmetry*, a concept often associated with order, beauty, and harmony. We can define **symmetry** as follows: An object exhibits symmetry when certain operations can be performed on it without changing its appearance. Consider the equilateral triangle in Figure 1.4a. If you close your eyes and someone rotates the triangle by 120° while you have your eyes closed, the triangle appears

Figure 1.4(a) Rotational symmetry: Rotating an equilateral triangle by 120° doesn't change how it looks

(b) Reflection symmetry: Across each reflection axis (labeled R), two sides of the triangle are mirror images of each other

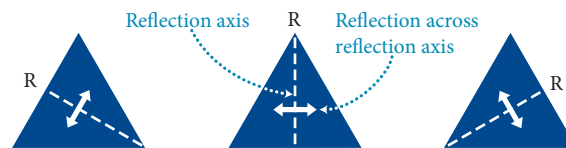
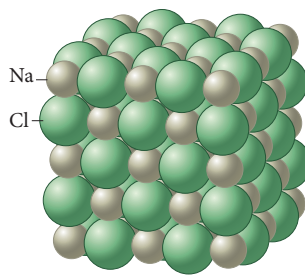


Figure 1.5 The symmetrical arrangement of atoms in a salt crystal gives these crystals their cubic shape.

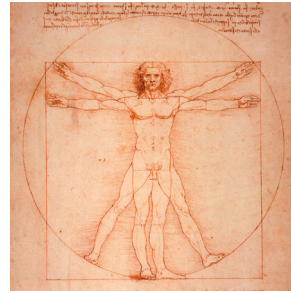
(a) Micrograph of salt crystals



(b) Symmetrical arrangement of atoms in a salt crystal



(c) Da Vinci's Vitruvian Man shows the reflection symmetry of the human body



the same when you open your eyes, and you can't tell that it has been rotated. The triangle is said to have *rotational symmetry*, one of several types of geometrical symmetry.

Another common type of geometrical symmetry, *reflection symmetry*, occurs when one half of an object is the mirror image of the other half. The equilateral triangle in Figure 1.4 possesses reflection symmetry about the three axes shown in Figure 1.4b. If you imagine folding the triangle in half over each axis, you can see that the two halves are identical. Reflection symmetry occurs all around us: in the arrangement of atoms in crystals (Figure 1.5a and b) and in the anatomy of most life forms (Figure 1.5c), to name just two examples.

The ideas of symmetry—that something appears unchanged under certain operations—apply not only to the shape of objects but also to the more abstract realm of physics. If there are things we can do to an experiment that leave the result of the experiment unchanged, then the phenomenon tested by the experiment is said to possess certain symmetries. Suppose we build an apparatus, carry out a certain measurement in a certain location, then move the apparatus to another location, repeat the measurement, and get the same result in both locations.* By moving the apparatus to a new location (*translating* it) and obtaining the same result, we have shown that the observed phenomenon has *translational symmetry*. Any physical law that describes this phenomenon must therefore mathematically exhibit translational symmetry; that is, the mathematical expression of this law must be independent of the location.

Likewise, we expect any measurements we make with our apparatus to be the same at a later time as at an earlier time; that is, translation in time has no effect on the measurements. The laws describing the phenomenon we are

*In moving our apparatus, we must take care to move any relevant external influences along with it. For example, if Earth's gravity is of importance, then moving the apparatus to a location in space far from Earth does not yield the same result.

studying must therefore mathematically exhibit symmetry under translation in time; in other words, the mathematical expression of these laws must be independent of time.

Exercise 1.3 Change is no change

Figure 1.6 shows a snowflake. Does the snowflake have rotational symmetry? If yes, describe the ways in which the flake can be rotated without changing its appearance. Does it have reflection symmetry? If yes, describe the ways in which the flake can be split in two so that one half is the mirror image of the other.

Figure 1.6 Exercise 1.3.

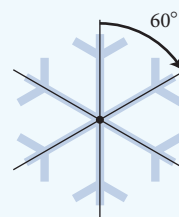


SOLUTION I can rotate the snowflake by 60° or a multiple of 60° (120° , 180° , 240° , 300° , and 360°) in the plane of the photograph without changing its appearance (Figure 1.7a). It therefore has rotational symmetry.

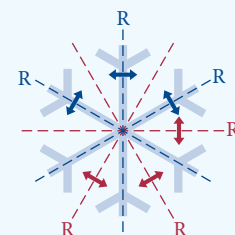
I can also fold the flake in half along any of the three blue axes and along any of the three red axes in Figure 1.7b. The flake therefore has reflection symmetry about all six of these axes.

Figure 1.7

(a) Rotational symmetry



(b) Reflection symmetry



A number of such symmetries have been identified, and the basic laws that govern the inner workings of the physical world must reflect these symmetries. Some of these symmetries are familiar to you, such as translational symmetry in space or time. Others, like electrical charge or parity symmetry, are unfamiliar and surprising and go beyond the scope of this course. Whereas symmetry has always implicitly been applied to the description of the universe, it plays an increasingly important role in physics: In a sense the quest of physics in the 21st century is the search for (and test of) symmetries because these symmetries are the most fundamental principles that all physical laws must obey.



1.4 You always store your pencils in a cylindrical case. One day while traveling in the tropics, you discover that the cap, which you have placed back on the case day in, day out for years, doesn't fit over the case. What do you conclude?

1.3 Matter and the universe

The goal of physics is to describe all that happens in the universe. Simply put, the **universe** is the totality of matter and energy combined with the space and time in which all events happen—everything that is directly or indirectly observable. To describe the universe, we use *concepts*, which are ideas or general notions used to analyze natural phenomena.* To provide a quantitative description, these concepts must be expressed quantitatively, which requires defining a procedure for measuring them. Examples are the length or mass of an object, temperature, and time intervals. Such **physical quantities** are the cornerstones of physics. It is the accurate measurement of physical quantities that has led to the great discoveries of physics. Although many of the fundamental concepts we use in this book are familiar ones, quite a few are difficult to define in words, and we must often resort to defining these concepts in terms of the procedures used to measure them.

The fundamental physical quantity by which we map out the universe is **length**—a distance or an extent in space. The length of a straight or curved line is measured by comparing the length of the line with some standard length. In 1791, the French Academy of Sciences defined the standard unit for length, called the **meter** and abbreviated m, as one ten-millionth of the distance from the equator to the North Pole. For practical reasons, the standard was redefined in 1889 as the distance between two fine lines engraved on a bar of platinum-iridium alloy kept at the International Bureau of Weights and Measures near Paris. With the advent of lasers, however, it became possible to measure the speed of light with extraordinary accuracy, and so the meter was redefined in 1983 as the distance traveled

by light in vacuum in a time interval of $1/299,792,458$ of a second. This number is chosen so as to make the speed of light exactly 299,792,458 meters per second and yield a standard length for the meter that is very close to the length of the original platinum-iridium standard. This laser-based standard is final and will never need to be revised.



1.5 Based on the early definition of the meter, one ten-millionth of the distance from the equator to the North Pole, what is Earth's radius?

Now that we have defined a standard for length, let us use this standard to discuss the structure and size scales of the universe. Because of the extraordinary range of size scales in the universe, we shall round off any values to the nearest power of ten. Such a value is called an **order of magnitude**. For example, any number between 0.3 and 3 has an order of magnitude of 1 because it is within a factor of 3 of 1; any number greater than 3 and equal to or less than 30 has an order of magnitude of 10. You determine the order of magnitude of any quantity by writing it in scientific notation and rounding the coefficient in front of the power of ten to 1 if it is equal to or less than 3 or to 10 if it is greater than 3.[†] For example, 3 minutes is 180 s, which can be written as 1.8×10^2 s. The coefficient, 1.8, rounds to 1, and so the order of magnitude is 1×10^2 s = 10^2 s. The quantity 680, to take another example, can be written as 6.8×10^2 ; the coefficient 6.8 rounds to 10, and so the order of magnitude is 10×10^2 = 10^3 . And Earth's circumference is 40,000,000 m, which can be written as 4×10^7 m; the order of magnitude of this number is 10^8 m. You may think that using order-of-magnitude approximations is not very scientific because of the lack of accuracy, but the ability to work effectively with orders of magnitude is a key skill not just in science but also in any other quantitative field of endeavor.

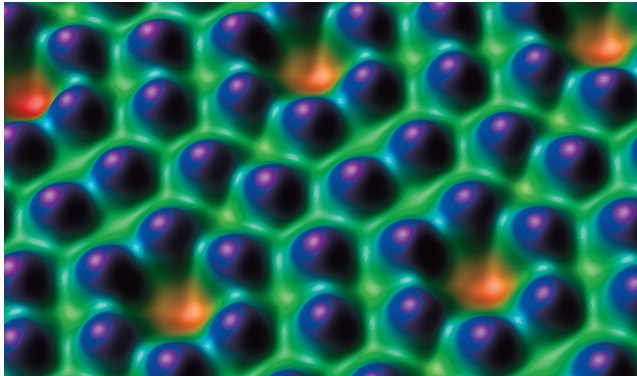
All ordinary matter in the universe is made up of basic building blocks called **atoms** (Figure 1.8). Nearly all the matter in an atom is contained in a dense central nucleus, which consists of **protons** and **neutrons**, two types of subatomic particles. A tenuous *cloud* of **electrons**, a third type of subatomic particle, surrounds this nucleus. Atoms are spherical and have a diameter of about 10^{-10} m. Atomic nuclei are also spherical, with a diameter of about 10^{-15} m, making atoms mostly empty space. Atoms attract one another when they are a small distance apart but resist being squeezed into one another. The arrangement of atoms in a material determines the properties of the material.

Figure 1.9 shows the relative size of some representative objects in the universe. The figure reveals a lot about the organization of matter in the universe and serves as a visual model of the structure of the universe. Roughly

*When an important concept is introduced in this book, the main word pertaining to the concept is printed in **boldface type**. All important concepts introduced in a chapter are listed at the end of the chapter, in the Chapter Glossary.

[†]The reason we use 3 in order-of-magnitude rounding, and not 5 as in ordinary rounding, is that orders of magnitude are logarithmic, and on this logarithmic scale $\log 3 = 0.48$ lies nearly halfway between $\log 1 = 0$ and $\log 10 = 1$.

Figure 1.8 Scanning tunneling microscope image showing the individual atoms that make up a silicon surface. The size of each atom is about 1/50,000 the width of a human hair.



speaking, there is clustering of matter from smaller to larger at four length scales. At the subatomic scale, most of the matter in an atom is compressed into the tiny atomic nucleus, a cluster of subatomic particles. Atoms, in turn, cluster to form the objects and materials that surround us, from viruses to plants, animals, and other everyday objects. The next level is the clustering of matter in stars, some of which, such as the Sun, are surrounded by planets like Earth. Stars, in turn cluster to form galaxies. As we shall discuss in Chapter 7, this clustering of matter reveals a great deal about the way different objects interact with one another.

Exercise 1.4 Tiny universe

If all the matter in the observable universe were squeezed together as tightly as the matter in the nucleus of an atom, what order of magnitude would the diameter of the universe be?

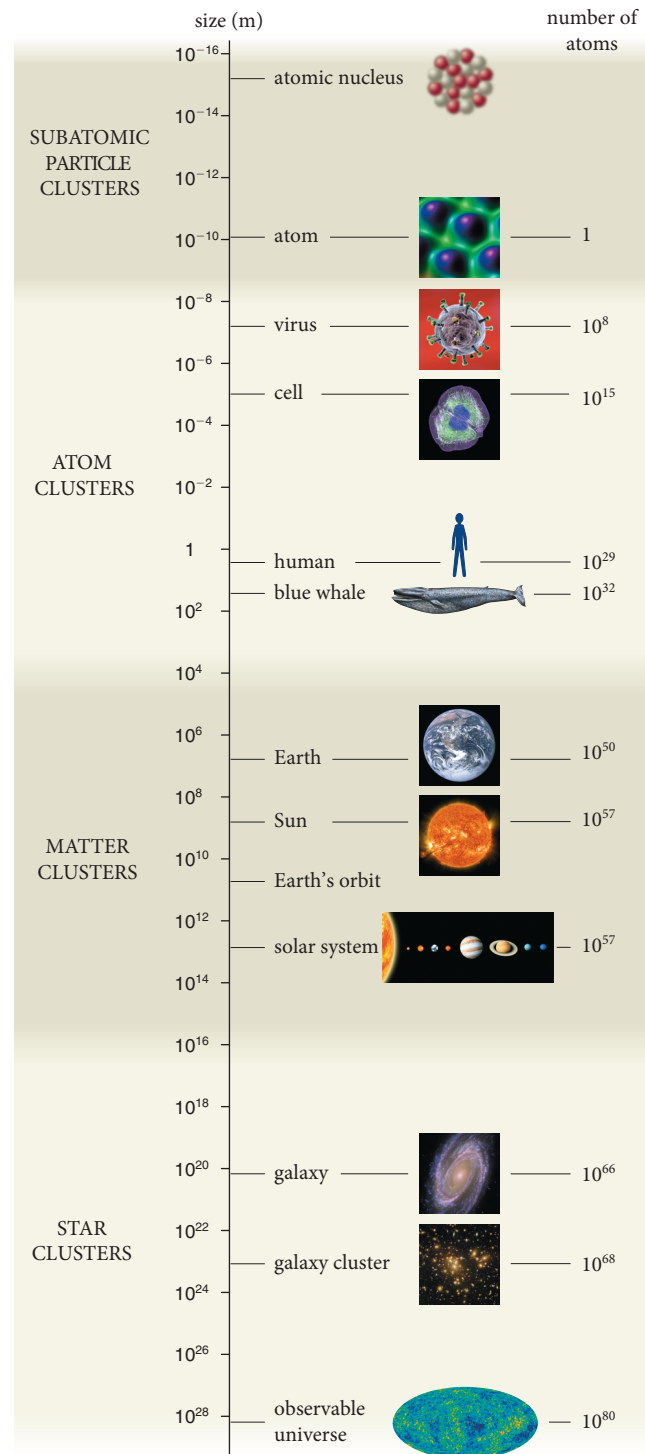
SOLUTION From Figure 1.9 I see that there are about 10^{80} atoms in the universe. I can arrange these atoms in a cube that has 10^{27} atoms on one side because such a cube could accommodate $10^{27} \times 10^{27} \times 10^{27} = 10^{81}$ atoms. Given that the diameter of a nucleus is about 10^{-15} m, the length of a side of this cube would be

$$(10^{27} \text{ atoms})(10^{-15} \text{ m per atom}) = 10^{12} \text{ m,}$$

which is a bit larger than the diameter of Earth's orbit around the Sun.

An alternative method for obtaining the answer is to realize that the matter in a single nucleus occupies a cubic volume of about $(10^{-15} \text{ m})^3 = 10^{-45} \text{ m}^3$. If all the matter in the universe were squeezed together just as tightly, it would occupy a volume of about 10^{80} times the volume of an atomic nucleus, or $10^{80} \times 10^{-45} \text{ m}^3 = 10^{35} \text{ m}^3$. The side of a cube of this volume is equal to the cube root of 10^{35} m^3 , or $4.6 \times 10^{11} \text{ m}$, which is the same order of magnitude as my first answer.

Figure 1.9 A survey of the size and structure of the universe.



CONCEPTS



1.6 Imagine magnifying each atom in an apple to the size of the apple. What would the diameter of the apple then be?

1.4 Time and change

Profound and mysterious, time is perhaps the greatest enigma in physics. We all know what is meant by *time*, but it is difficult, if not impossible, to explain the idea in words. (Put the book down for a minute and try defining *time* in words before reading on.) One way to describe time is that it is the infinite continual progression of events in the past, present, and future, often experienced as a force that moves the world along. This definition is neither illuminating nor scientifically meaningful because it merely relates the concept of time to other, even less well-defined notions. Time is defined by the rhythm of life, by the passing of days, by the cycle of the seasons, by birth and death. However, even though many individual phenomena, such as the 24-hour cycle of the days, the cycle of seasons, and the swinging of a pendulum, are repetitive, the time we experience does not appear to be repetitive, and the current view is that time is a continuous succession of events.

The irreversible flow of time controls our lives, pushing us inexorably forward from the past to the future. Whereas we can freely choose our location and direction in all three dimensions in space, time flows in a single direction, dragging us forward with it. Time thus presents less symmetry than the three dimensions of space: Although opposite directions in space are equivalent, opposite directions in time are not equivalent. The “arrow of time” points only into the future, a direction we define as the one we have no memory of. Curiously, most of the laws of physics have no requirement that time has to flow in one direction only, and it is not until Chapter 19 that we can begin to understand why events in time are irreversible.

The arrow of time allows us to establish a *causal relationship* between events. For example, lightning causes thunder and so lightning has to occur *before* the thunder. This statement is true for all observers: No matter who is watching the storm and no matter where that storm is happening, every observer first sees a lightning bolt and only after that hears the thunder because an effect never precedes its cause. Indeed, the very organization of our thoughts depends on the **principle of causality**:

Whenever an event A causes an event B, all observers see event A happening first.

Without this principle, it wouldn't be possible to develop any scientific understanding of how the world works. (No physics course to take!) The principle of causality also makes it possible to state a definition: **Time** is a physical quantity that allows us to determine the sequence in which related events occur.

To apply the principle of causality and sort out causes and effects, it is necessary to develop devices—clocks—for keeping track of time. All clocks operate on the same principle: They repeatedly return to the same state. The rotation of Earth about its axis can serve as a clock if we note

the instant the Sun reaches its highest position in the sky on successive days. A swinging pendulum, which repeatedly returns to the same vertical position, can also serve as a clock. The time interval between two events can be determined by counting the number of pendulum swings between the events. The accuracy of time measurements can be increased by using a clock that has a large number of repetitions in a given time interval.



1.7 (a) State a possible cause for the following events: (i) The light goes out in your room; (ii) you hear a loud, rumbling noise; (iii) a check you wrote at the bookstore bounces. (b) Could any of the causes you named have occurred after their associated event? (c) Describe how you feel when you experience an event but don't know what caused it—you hear a strange noise when camping, for instance, or an unexpected package is sitting on your doorstep.

The familiar standard unit for measuring time is the **second** (abbreviated s), originally defined as $1/86,400$ of a day but currently more accurately defined as the duration of 9,192,631,770 periods of certain radiation emitted by cesium atoms. **Figure 1.10** gives an idea of the vast range of time scales in the universe.

The English physicist Isaac Newton stated, “Absolute, true, and mathematical time, of itself and from its own nature, flows equably without relation to anything external.” In other words, the notion of past, present, and future is universal—“now” for you, wherever you are at this instant, is also “now” everywhere else in the universe. Although this notion of the universality of time, which is given the name **absolute time**, is intuitive, experiments described in Chapter 14 have shown this notion to be false. Still, for many experiments and for the material we discuss in most of this book, the notion of absolute time remains an excellent approximation.

Now that we have introduced space and time, we can use these concepts to study events. Throughout this book, we focus on **change**, the transition from one state to another. Examples of change are the melting of an ice cube, motion (a change in location), the expansion of a piece of metal, the flow of a liquid. As you will see, one might well call physics the study of the changes that surround us and convey the passage of time. What is most remarkable about all this is that we shall discover that underneath all the changes we'll look at, certain properties remain *unchanged*. These properties give rise to what are called *conservation laws*, the most fundamental and universal laws of physics.

There is a profound aesthetic appeal in knowing that symmetry and conservation are the cornerstones of the laws that govern the universe. It is reassuring to know that an elegant simplicity underlies the structure of the universe and the relationship between space and time.


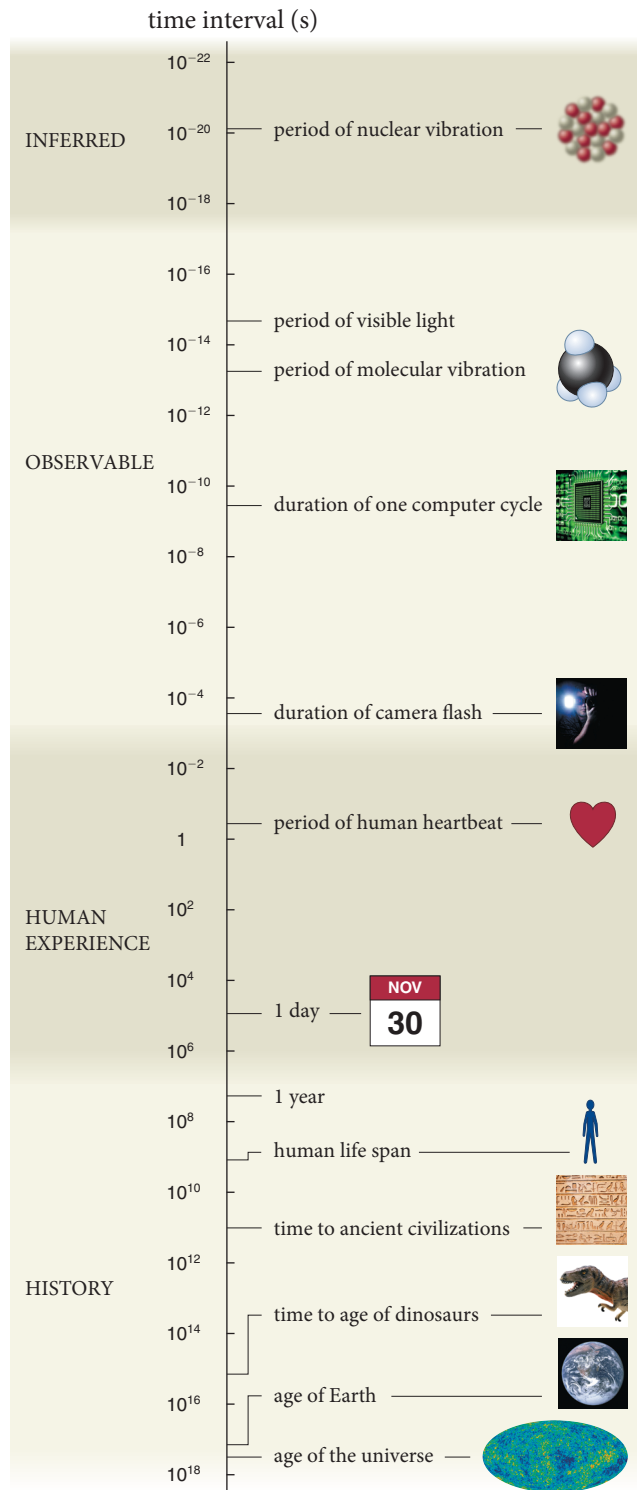

 **1.8** A single chemical reaction takes about 10^{-13} s. What order of magnitude is the number of sequential chemical reactions that could take place during a physics class?

Figure 1.10 A survey of the time scales on which events in the universe take place.



1.5 Representations

Of all our senses, it is vision that most informs our mind. For this reason, expert problem solvers rarely start working on a problem without first making some sort of visual representation of the available information, and you should always do the same. Such a drawing helps you establish a clear mental image of the situation, interpret the problem in light of your own knowledge and experience, develop a qualitative understanding of the problem, and organize the information in a meaningful way. Without the drawing, you have to juggle all the information in your head. The drawing frees your brain to deal with the solution. As an example, consider the situation described in Checkpoint 1.9. First try solving the problem without a piece of paper and note the mental effort it takes; then make a sketch and work out the solution.

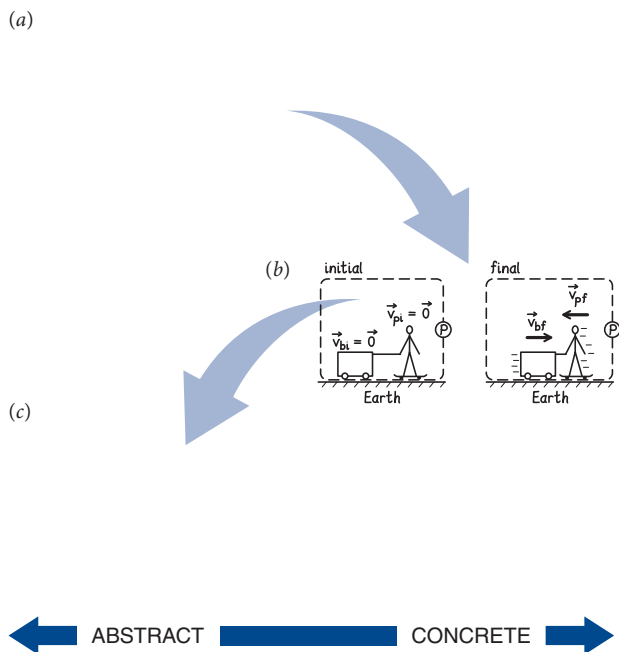
 **1.9** You and your spouse are working out a seating arrangement around a circular table for dinner with John and Mary Jones, Mike and Sylvia Masters, and Bob and Cyndi Ahlers. Mike is not fond of the Ahlers, and Sylvia asked that she not be seated next to John. You would like to alternate men and women and avoid seating spouses next to each other. Determine an arrangement that satisfies all the constraints.

Were you able to solve the problem without a sketch? Doing so would be difficult because there is more information than most people can comfortably keep in mind at once. When you represent the information visually, however, it is relatively easy to solve this problem (see the solution in the back of this book). The drawing breaks the problem down into small steps and helps you articulate in your mind what you are trying to accomplish.

Visual representations are not helpful only for making seating arrangements. They work just as well for solving physics problems, although it may not immediately be clear *how* to represent the available information. For this reason we shall develop a number of different ways—pictures, sketches, diagrams, graphs—and a number of different context-specific procedures to represent information visually in our study of physics. As you will see, these visual representations are an integral part of getting a grip on a problem and developing a model (Figure 1.11).

As we discussed in Section 1.1, descriptions of the physical world always begin with simplified representations. When you solve physics problems, elaborate drawings clutter your mind with irrelevant information and prevent you from getting a clear view of the important features. One of the most basic skills in physics, therefore, is to decide what to leave out of your drawings. If you leave out essential features, the representation is useless, but if you put in too many details, it becomes impossible to analyze the situation. Sometimes it is necessary to begin with an oversimplification in order to develop a feel for a given situation. Once this initial understanding has been gained, however,

Figure 1.11 Multiple representations help you solve problems. (a) Many problems start with a verbal representation. (b) Turning the words into a sketch helps you to grasp the problem. (c) The sketch can then give meaning to a mathematical representation of the problem.

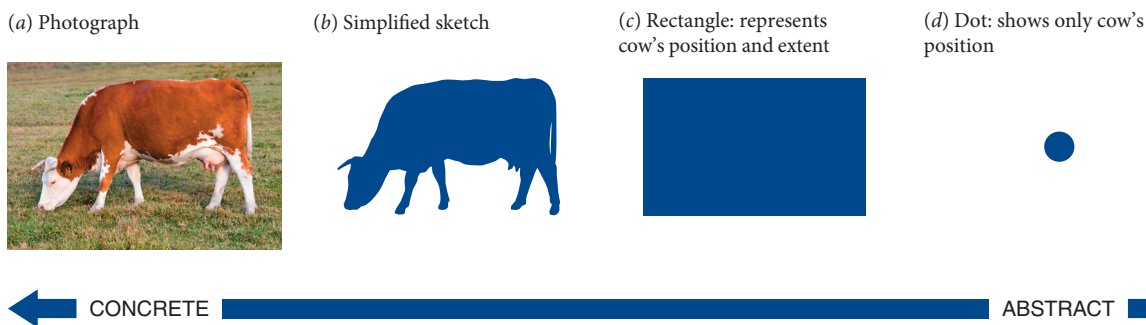


it becomes possible to construct less idealized models, with each successive model being a more realistic representation of the real-world situation.

In any drawings you make, therefore, you should treat everyday objects as simplifications that can be characterized by a minimum number of features or quantities. Some joke that physicists hold a grossly oversimplified view of the world, thinking in such terms as, for instance, “Consider a spherical cow...” The world around us, cows included, contains infinitely many details that may play a role in the grand scheme of things, but to get a grip on any problem it is important to begin by leaving out as much detail as possible. If the resulting model reproduces the main features of the real world, you know you have taken the essential

CONCEPTS

Figure 1.12 Increasingly simplified and abstract representations of a cow.



attributes into account. As an example, **Figure 1.12** shows a progression from a photograph of a cow to an abstract rendition of it. To study the pattern on the cow’s hide, you need the photograph. If you are interested in only the position of a certain cow in a certain pasture, however, a simple dot suffices (**Figure 1.12d**). By reducing the cow to a dot, you have discarded any information about its size and shape, but as you will see as you continue with your study of physics, this information is often not relevant.

Exercise 1.5 Stretching a spring

For a physics laboratory assignment, one end of a spring is attached to a horizontal rod so that the spring hangs vertically, and a ruler is hung vertically alongside the spring. The stretching properties of the spring are to be measured by attaching eight identical beads to the spring’s free end. With no beads attached, the free end of the spring is at a ruler reading of 23.4 mm. With one bead attached, the end of the spring drops to 25.2 mm. When the second, third, and fourth beads are attached one at a time, the end drops to ruler readings of 26.5 mm, 29.1 mm, and 30.8 mm, respectively. Adding the fifth and sixth beads together moves the spring end to 34.3 mm, and adding the last two beads moves the end to 38.2 mm. (a) Make a pictorial representation of this setup. (b) Tabulate the data. (c) Plot the data on a graph, showing the ruler readings on the vertical axis and the numbers of beads on the horizontal axis. (d) Describe what can be inferred from the data.

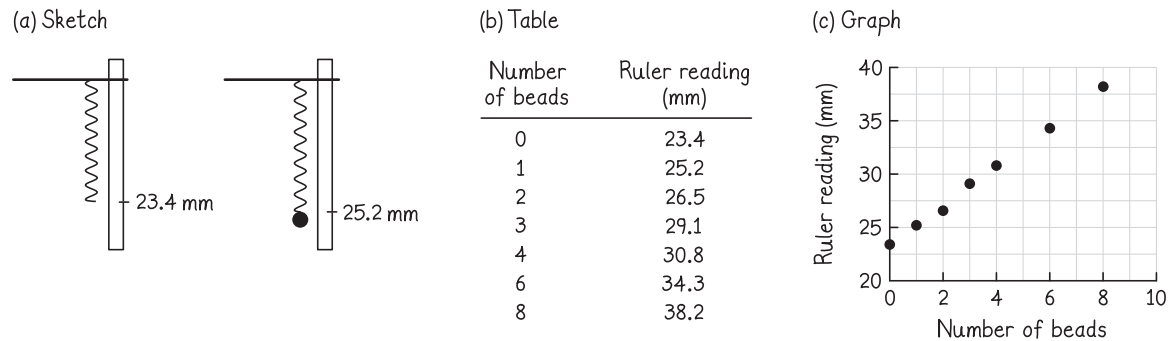
SOLUTION (a) The important items that should appear in my drawing are the spring, the rod from which it is suspended, the ruler, and at least one bead. My drawing should also indicate how the ruler readings are obtained. I can illustrate this procedure with just one bead, as **Figure 1.13a** shows. By drawing the spring first with no beads attached and then with one bead attached and showing the two ruler readings, I’ve represented the general procedure of adding beads one (or two) at a time and paying attention to how each addition changes the position of the spring end.

(b) See **Figure 1.13b**.

(c) See **Figure 1.13c**.

(d) The relationship between the ruler readings and the numbers of suspended beads is linear. That is to say, each additional bead stretches the spring by about the same amount.

Figure 1.13



Exercise 1.5 demonstrates how various ways of representing information help you interpret data. The pictorial representation of Figure 1.13a helps you visualize how the measurements were taken. The table of Figure 1.13b organizes the data, and the graph of Figure 1.13c allows you to recognize the linear relationship between how far the spring stretches and the number of beads suspended from it—something that is not at all obvious from the text or the table.

Like the sketch, the graph is a simplified representation of the stretching of the spring. Each representation involves a loss of information and detail. The sketch is a simplified two-dimensional representation of a three-dimensional setup, and the graph shows only one piece of information for each measurement: the position of the bottom of the spring. All other information is left out in order to reveal one crucial point: How much the spring stretches is proportional to the number of beads suspended from it (something we look at in more detail in Chapter 9).

You may be beginning to wonder what the role of mathematics is in physics, given that we haven't used any thus far. One of the main roles of mathematics in physics is allowing us to express succinctly and unambiguously ideas that if expressed verbally would require a lot of words whose meaning may not be precise. Take this statement, for example:

The magnitude of the acceleration of an object is directly proportional to the magnitude of the vector sum of the forces exerted on the object and inversely proportional to the object's inertia. The direction of the acceleration is the same as the direction of the vector sum of the forces.

(Don't worry about the meaning of this statement for now; just note that it is quite a mouthful.) We can express this

statement mathematically as

$$\vec{a} = \frac{\sum \vec{F}}{m}.$$

These two expressions—one verbal, the other mathematical—are identical. Neither one is more accurate than the other. Right now the symbols in the equation may make no more sense to you than hieroglyphs. Once you understand their meaning, however, the mathematical expression can be parsed much more quickly than the verbal expression can. In this respect, mathematics plays the same role as visual representations: It relieves the brain of having to keep track of many words. Another important benefit of mathematical representation is that we can use the techniques of mathematics to manipulate the symbols and obtain new insights.

Without an understanding of the meaning of the concepts in any expression (acceleration and force in our example here), however, verbal expressions and mathematical ones are both meaningless, and so it is important to focus first on the meanings of concepts. As you will notice, this book has been designed to develop concepts first and to emphasize the visual representation of these concepts before moving on to a mathematical treatment (see the box “Organization of this book”).



1.10 Picture a long, straight corridor running east-west, with a water fountain located somewhere along it. Starting from the west end of the corridor, a woman walks a short distance east along the corridor and stops before reaching the water fountain. The distance from her to the fountain is twice the distance she has walked. She then continues walking east, passes the water fountain, and stops 60 m from her first stop. Now the distance from her to the fountain is twice the distance from her to the east end of the corridor. How long is the corridor?

Organization of this book

The material for this course is presented in two volumes. The first volume (the one you are reading now, *Principles*) is aimed at guiding you in developing a solid understanding of the principles of physics. The second volume, *Practice*, provides a broad variety of questions and problems that allow you to apply and sharpen your understanding of physics.

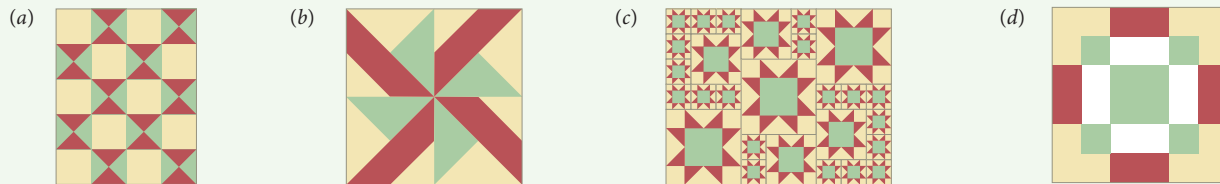
Each chapter in the *Principles* volume is divided into two parts: *Concepts* and *Quantitative Tools*. The *Concepts* part develops the conceptual framework for the subject of the chapter, concentrating on the underlying ideas and helping you develop a mental picture of the subject. The *Quantitative Tools* part develops the mathematical framework, building on the ideas developed in the *Concepts* part. Interspersed in the text of each chapter are several types of learning aids to guide you through the chapter:

1. **Checkpoints** (👉). These questions compel you to test yourself on how well you understand the material you just read. Do not skip them as you read the text. First of all, the answer to a checkpoint may be necessary to understand the text following that checkpoint. Second, and more important, I've put these checkpoints right in the text because working on them means learning the material. The answers to all the checkpoints are at the end of this volume.
2. **Exercises and examples.** The fully-worked-out exercises and examples help you develop and apply problem-solving strategies. It is generally a good idea to attempt to solve the problem by yourself before reading the solution. More information on general approaches to problem solving is given in Section 1.8.
3. **Procedures.** Approaches for analyzing specific situations are given in separate, highlighted boxes.
4. **Self-quiz.** The Self-quiz in each *Principles* chapter, which always comes at the end of the conceptual part of the chapter, allows you to assess your understanding of the concepts before you move on to the quantitative treatment. Complete each Self-quiz before working on the quantitative part of the chapter, even if you are already familiar with most of the material covered. Before tackling the quantitative material, be sure to resolve any difficulties you might have in answering a Self-quiz question, either by rereading the material in the conceptual part of the chapter or by consulting your instructor.
5. **Glossary.** At the end of each chapter is a list defining the important concepts in the chapter (which are the terms printed in bold).

Self-quiz

- Two children in a playground swing on two swings of unequal length. The child on the shorter swing is considerably heavier than the child on the longer swing. You observe that the longer swing swings more slowly. Formulate a hypothesis that could explain your observation. How could you test your hypothesis?
- What symmetries do you observe in the quilt patterns of **Figure 1.14**?

Figure 1.14

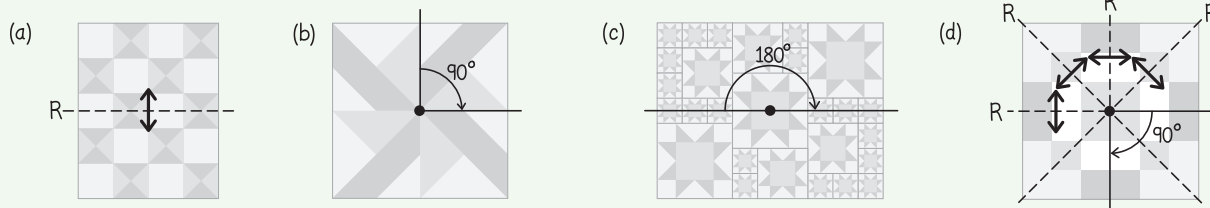


- Give the order of magnitude of these quantities in meters or seconds: (a) length of a football field, (b) height of a mature tree, (c) one week, (d) one year.
- Starting from the first floor, an elevator stops at floors 5, 2, 4, 3, 6, and 4 before returning to the first floor. (a) Represent this motion visually. (b) If the distance between the first and sixth floors is 15 m, what is the total distance traveled by the elevator?

Answers

- One hypothesis is that longer swings swing more slowly than shorter swings. You can test this hypothesis by adjusting the length of either swing until the two lengths are the same and then asking the children to remount their respective swings and swing again. If the originally longer swing is still the slower one, your hypothesis is not correct. If the two swings now have the same speed, your hypothesis is correct. Another hypothesis is that heavier children swing faster than lighter ones. You can test this hypothesis by asking the children to trade places. If the longer swing now swings faster than the shorter swing, your hypothesis is correct. If the longer swing still swings more slowly, your hypothesis is incorrect.
- See **Figure 1.15**. (a) Reflection symmetry about a horizontal line through the center. (b) Rotational symmetry by multiples of 90° . (c) Rotational symmetry by 180° . (d) Rotational symmetry by multiples of 90° and reflection symmetry about a horizontal, vertical, or diagonal line through the center.

Figure 1.15



- (a) 100 yards is about 100 m; the order of magnitude is thus $100 \text{ m} = 10^2 \text{ m}$. (b) An average mature tree is between 5 and 20 m tall, for an average of $12 \text{ m} = 1.2 \times 10^1 \text{ m}$. The coefficient 1.2 rounds to 1, and so the order of magnitude is $1 \times 10^1 \text{ m} = 10 \text{ m}$. (c) $1 \text{ week} = (1 \text{ week})(7 \text{ days/week})(24 \text{ h/day})(60 \text{ min/h})(60 \text{ s/min}) = 604,800 \text{ s} = 6 \times 10^5 \text{ s}$; the coefficient 6 rounds to 10, and so the order of magnitude is $10 \times 10^5 \text{ s} = 10^6 \text{ s}$. (d) $1 \text{ year} = 52 \text{ weeks} = (52 \text{ weeks})(604,800 \text{ s/week}) = 31,449,600 \text{ s} = 3.1 \times 10^7 \text{ s}$; the coefficient 3.1 rounds to 10, and so the order of magnitude is $10 \times 10^7 \text{ s} = 10^8 \text{ s}$.
- (a) See **Figure 1.16** for one way to represent the motion. Note that the elevator itself is not represented because showing it would add nothing we need to the visual information. The only thing we are interested in is distances traveled, represented by the vertical lines. (b) If the distance between floors 1 and 6 is 15 m, one floor is $(15 \text{ m})/5 = 3.0 \text{ m}$. From the figure, I see that the numbers of floors traveled between successive stops are 4, 3, 2, 1, 3, 2, and 3, for a total of 18 floors, or $18(3.0 \text{ m}) = 54 \text{ m}$.

Figure 1.16

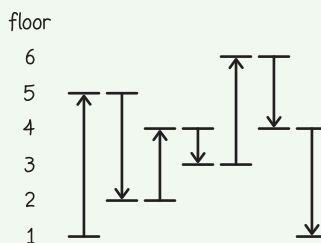


Table 1.1 Physical quantities and their symbols

| Physical quantity | Symbol |
|-------------------|--------|
| length | ℓ |
| time | t |
| mass | m |
| speed | v |
| volume | V |
| energy | E |
| temperature | T |

1.6 Physical quantities and units

Because physics is a quantitative science, statements must be expressed in numbers, which requires either measuring or calculating numerical values for physical quantities. In this section we review some basic rules for dealing with physical quantities, which in this book are represented by italic symbols—typically letters from the Roman or Greek alphabet, such as t for time and σ for electrical conductivity. **Table 1.1** gives the symbols for some of the physical quantities we use throughout the book.

Physical quantities are expressed as the product of a numerical value and a unit of measurement. For example, the length ℓ of an object that is 1.2 m long can be expressed as $\ell = 1.2 \text{ m}$. The unit system used in science and engineering throughout the world and in everyday life in most countries is the *Système International* (International System), and the units are collectively called **SI units**. This system consists of seven base units (**Table 1.2**) from which all other units currently in use can be derived. For example, the physical quantity speed, which we discuss in Chapter 2, is defined as the distance traveled divided by the time interval over which the travel takes place. Thus the SI derived unit of speed is meters per second (m/s), the base unit of length divided by the base unit of time. A list of SI derived units and their relationship to the seven base units is given in Appendix C.

Be careful not to confuse abbreviations for units with symbols for physical quantities. Unit abbreviations are printed in roman (upright) type—m for meters, for instance—and symbols for physical quantities are printed in italic (slanted) type— t for time, say. Also, bear in mind that you can add and subtract quantities only if they have the same units; it is meaningless to add, say, 3 m to 4 kg.

To produce multiples of any SI unit and conveniently work with very large or very small numbers, we modify the unit name with prefixes representing integer powers of ten (**Table 1.3**). For example, a billionth of a second is denoted by 1 ns (pronounced “one nanosecond”):

$$1 \text{ ns} = 10^{-9} \text{ s.} \quad (1.1)$$

One thousand meters is denoted by 1 km, “one kilometer.” Prefixes are never used without a unit and are never combined into compound prefixes. The unit *kilogram* contains a prefix (*kilo-*) because it is derived from the non-SI unit *gram* (1 kg = 1000 g). Therefore 10^{-6} kg never becomes $1 \mu\text{kg}$. Instead, the names and multiples of the kilogram are constructed by adding the appropriate prefix to the word *gram* and the abbreviation g. For example, 10^{-6} kg becomes 1 mg, “one milligram.”

The standard practice in engineering is to use only the powers of ten that are multiples of three.

Table 1.2 The seven SI base units

| Name of unit | Abbreviation | Physical quantity |
|--------------|--------------|---------------------------|
| meter | m | length |
| kilogram | kg | mass |
| second | s | time |
| ampere | A | electric current |
| kelvin | K | thermodynamic temperature |
| mole | mol | amount of substance |
| candela | cd | luminous intensity |

Table 1.3 SI prefixes

| 10^n | Prefix | Abbreviation | 10^n | Prefix | Abbreviation |
|-----------|--------|--------------|------------|--------|--------------|
| 10^0 | — | — | | | |
| 10^3 | kilo- | k | 10^{-3} | milli- | m |
| 10^6 | mega- | M | 10^{-6} | micro- | μ |
| 10^9 | giga- | G | 10^{-9} | nano- | n |
| 10^{12} | tera- | T | 10^{-12} | pico- | p |
| 10^{15} | peta- | P | 10^{-15} | femto- | f |
| 10^{18} | exa- | E | 10^{-18} | atto- | a |
| 10^{21} | zetta- | Z | 10^{-21} | zepto- | z |
| 10^{24} | yotta- | Y | 10^{-24} | yocto- | y |



1.11 Use prefixes from Table 1.3 to remove either all or almost all of the zeros in each expression. (a) $\ell = 150,000,000$ m, (b) $t = 0.000\,000\,000\,012$ s, (c) 1200 km/s, (d) 2300 kg.

Of the seven SI base units, we have already discussed two, the meter and the second. We discuss the base unit for mass, the kilogram, in Chapter 4, the base unit for electric current, the ampere, in Chapter 27, and the base unit for temperature, the kelvin, in Chapter 20.

The **mole** (abbreviated mol) is the SI base unit that measures the quantity of a given substance. A mole is currently defined as the number of atoms in 12×10^{-3} kg of carbon-12, the most common form of carbon. This number is called **Avogadro's number** N_A , after the 19th-century Italian physicist Amedeo Avogadro. The currently accepted experimental measurement of Avogadro's number is

$$N_A = 6.0221413 \times 10^{23}.$$

Note that the mole is simply a number: Just as *one dozen* means 12 of anything and *one gross* means 144 of anything, *one mole* means 6.022×10^{23} of anything. So 1 mol of helium atoms is 6.022×10^{23} helium atoms, and 1 mol of carbon dioxide molecules is 6.022×10^{23} carbon dioxide molecules.

The final SI base unit, the *candela*, measures luminous intensity. One candela (1 cd) is roughly the amount of light generated by a single candle; the light emitted by a 100-watt light bulb is about 120 cd. The definition of the candela takes into account how the human eye perceives the intensity of various colors and is therefore rather unwieldy. For this reason we do not use the candela in this book in the chapters dealing with light, concentrating instead on the amount of energy carried by light.

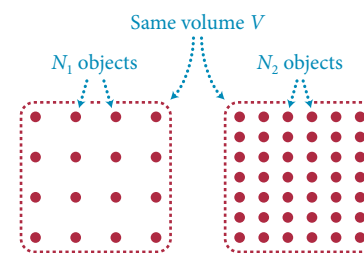
An important concept used throughout physics is **density**, the physical quantity that measures how much of some substance there is in a given volume. Depending on the quantity being measured, there are various types of density. For example, *number density* is the number of objects per unit volume. If there are N objects in a volume V , then the number density n of these objects is

$$n \equiv \frac{N}{V}. \quad (1.3)$$

(The symbol \equiv means that the equality is either a definition or a convention.) If the objects in a given volume are packed together more tightly, the number density is higher (Figure 1.17). *Mass density* ρ (Greek rho) is the amount of mass m per unit volume:

$$\rho \equiv \frac{m}{V}. \quad (1.4)$$

Figure 1.17 Number density.



The greater the number N of objects in a given space V , the higher the number density $n = N/V$. In this case $N_2 > N_1$, so $n_2 > n_1$.

Exercise 1.6 Helium density

At room temperature and atmospheric pressure, 1 mol of helium gas has a volume of $24.5 \times 10^{-3} \text{ m}^3$. The same amount of *liquid* helium has a volume of $32.0 \times 10^{-6} \text{ m}^3$. What are the number and mass densities of (a) the gaseous helium and (b) the liquid helium? The mass of one helium atom is $6.647 \times 10^{-27} \text{ kg}$.

SOLUTION (a) I know from Eq. 1.2 that 1 mol of helium contains 6.022×10^{23} atoms, and I can use this information in Eq. 1.3 to get the number density:

$$n = \frac{6.022 \times 10^{23} \text{ atoms}}{24.5 \times 10^{-3} \text{ m}^3} = 2.46 \times 10^{25} \text{ atoms/m}^3.$$

For the mass density, I must know the mass of 1 mol of helium atoms, and so I multiply the mass of one helium atom by the number of atoms in 1 mol:

$$\begin{aligned} m &= (6.647 \times 10^{-27} \text{ kg/atom})(6.022 \times 10^{23} \text{ atoms/mol}) \\ &= 4.003 \times 10^{-3} \text{ kg/mol.} \end{aligned}$$

Equation 1.4 then yields

$$\rho = \frac{4.003 \times 10^{-3} \text{ kg}}{24.5 \times 10^{-3} \text{ m}^3} = 0.163 \text{ kg/m}^3.$$

(b) For the liquid helium, the same reasoning gives me

$$\begin{aligned} n &= \frac{6.022 \times 10^{23} \text{ atoms}}{32.0 \times 10^{-6} \text{ m}^3} = 1.88 \times 10^{28} \text{ atoms/m}^3 \\ \rho &= \frac{4.003 \times 10^{-3} \text{ kg}}{32.0 \times 10^{-6} \text{ m}^3} = 125 \text{ kg/m}^3. \end{aligned}$$

Examples of non-SI units accepted for use along with SI units are the minute (1 min = 60 s), the hour (1 h = 3600 s), the liter (1 L = 10^{-3} m^3), and the metric ton (1 t = 10^3 kg).

A number of traditional, non-SI units are used in engineering; in various businesses, industries, sports, and trades; and in everyday life in the United States. Examples are inches, feet, yards, miles, acres, ounces, pints, gallons, and fluid ounces. These units are nondecimal, which makes it hard to interconvert them. When solving problems in this course, always begin by converting any quantities given in non-SI units to the SI equivalents. A conversion table is given in Appendix C.


The simplest way to convert from one unit to another is to write the conversion factor for the two units as a ratio. For, example, in Appendix C, we see that 1 in. = 25.4 mm. By bringing the 25.4 mm to the left side of the equals sign, we can write this as either

$$\frac{1 \text{ in.}}{25.4 \text{ mm}} = 1 \quad \text{or} \quad \frac{25.4 \text{ mm}}{1 \text{ in.}} = 1 \quad (1.5)$$

Note that you *must* write the units in these expressions because without them you obtain the incorrect expressions $\frac{1}{25.4} = 1$ and $\frac{25.4}{1} = 1$. Because multiplying any number by 1 doesn't change the number, you can use these ratios to convert

units. For example, to express 4.5 in. in millimeters, you multiply by the ratio on the right in Eq. 1.5 and cancel out the inches:

$$4.5 \text{ in.} = (4.5 \text{ in.}) \left(\frac{25.4 \text{ mm}}{1 \text{ in.}} \right) = 4.5 \times 25.4 \text{ mm} = 1.1 \times 10^2 \text{ mm.} \quad (1.6)$$

 **1.12** Why is the ratio on the left in Eq. 1.5 not suitable for converting inches to millimeters?

Exercise 1.7 Unit conversions

Convert each quantity to a quantity expressed either in meters or in meters raised to some power: (a) 4.5 in., (b) 3.2 acres, (c) 32 mi, (d) 3.0 pints.


SOLUTION I obtain my conversions factors from Appendix C.

$$(a) \quad (4.5 \text{ in.}) \left(\frac{2.54 \times 10^{-2} \text{ m}}{1 \text{ in.}} \right) = 1.1 \times 10^{-1} \text{ m.}$$

$$(b) \quad (3.2 \text{ acres}) \left(\frac{4.047 \times 10^3 \text{ m}^2}{1 \text{ acre}} \right) = 1.3 \times 10^4 \text{ m}^2.$$

$$(c) \quad (32 \text{ mi}) \left(\frac{1.609 \times 10^3 \text{ m}}{1 \text{ mi}} \right) = 5.1 \times 10^4 \text{ m.}$$

$$(d) \quad (3.0 \text{ pints}) \left(\frac{4.732 \times 10^{-4} \text{ m}^3}{1 \text{ pint}} \right) = 1.4 \times 10^{-3} \text{ m}^3.$$

 **1.13** (a) Using what you know about the diameters of atoms from Section 1.3, estimate the length of one side of a cube made up of 1 mol of closely packed carbon atoms. (b) The mass density of graphite (a form of carbon) is $2.2 \times 10^3 \text{ kg/m}^3$. By how much does the length you calculated in part a change when you do your calculation with this mass density value? Remember that 1 mol is the number of atoms in $12 \times 10^{-3} \text{ kg}$ of carbon.

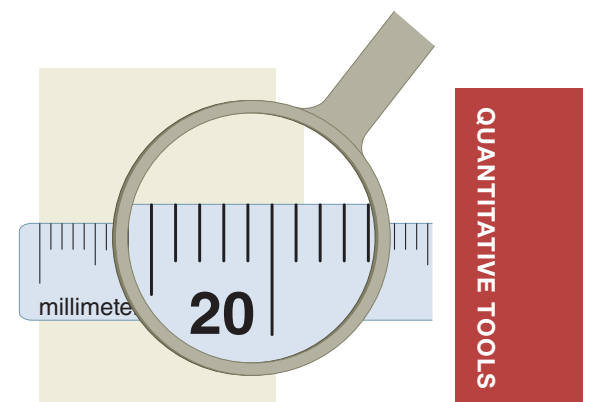
1.7 Significant digits

The numbers we deal with in physics fall into two categories: exact numbers that are known with complete certainty (integers, such as the 14 in “I have 14 books on my desk”) and inexact numbers that result from measurements and are known to only within some finite precision. Consider, for example, using a ruler to measure the width of a piece of paper (Figure 1.18). The width falls between the ruler marks for 21 mm and 22 mm and is closer to 21 mm than 22 mm. We might guess that the width is about 21.3 mm, but we cannot be sure about the last digit without a better measurement method. By recording the width as 21 mm, we are indicating that the actual value lies between 20.5 mm and 21.5 mm. The value 21 mm is said to have two **significant digits**—digits that are known reliably.

By expressing a value with the proper number of significant digits, we can convey the precision to which that value is known. For numbers that don't contain any zeros, all digits shown are significant, which means that 21 has two significant digits, as just noted, and 21.3 has three significant digits (implying that the actual value lies between 21.25 and 21.35).

With numbers that contain zeros, the situation is more complicated. *Leading zeros*, which means any that come before the first nonzero digit, are never significant: 0.037 has two significant digits. Zeros that come between two nonzero digits,

Figure 1.18 If you measure the width of a sticky note with the ruler shown, you can reliably read off two digits.



as in 0.602, are always significant. *Trailing zeros* are those that come after the last nonzero digit in a number, as in 3.500 and 20. Trailing zeros to the right of the decimal point are always significant: 25.10 has four significant digits. However, trailing zeros in numbers that do not contain a decimal point are ambiguous. For example, in the number 7900, it is not clear whether the trailing zeros are significant or not. The number of significant digits is at least two but could be three or four. To accurately convey the precision for such numbers, we use scientific notation, which lets us place significant zeros to the right of the decimal point: 7.900×10^3 has four significant digits, 7.90×10^3 has three, and 7.9×10^3 has two.

Important note: To simplify the notation in this book, we consider all trailing zeros in numbers that do not contain a decimal point to be significant: four significant digits for 3400, for instance, and two significant digits for 30.

Do not confuse significant digits with the number of decimal places or the number of digits: 0.000584 has three significant digits, six decimal places, and seven digits; 58.4 has three significant digits, one decimal place, and three digits. With the exception of fundamental constants such as the speed of light, most values in this book are given to two or three significant digits.

When you use a calculator for the math involved in solving physics problems, the calculator display usually shows many more digits than the number of significant digits allowed by the problem data. In such cases, you need to round your answer to the correct number of significant digits. If the digit just to the right of the last significant digit you are allowed is less than 5, report your last significant digit as it appears on the calculator display. If the digit just to the right of the last significant digit is 5 or greater, increase your last significant digit by 1. For example, if you are allowed two significant digits, 1.356 rounds to 1.4, 2.5199 rounds to 2.5, and 7.95 rounds to 8.0.

Multiple roundings can yield an accumulation of errors, and therefore it is best to wait until you have obtained the final result in a multistep calculation before rounding. In intermediate results, therefore, retain a few more digits than what your input quantities have and then round off to the correct number of significant digits only in your final result.

Exercise 1.8 Significant digits

(a) How many significant digits are there in 403.54 kg, 3.010×10^{57} m, 2.43×10^{-3} s, $14.00 \mu\text{m}$, 0.0140 s, 5300 kg? (b) Round 12,300 kg and 0.0125 s to two significant digits.

SOLUTION (a) 403.54 kg has five, 3.010×10^{57} m has four, 2.43×10^{-3} s has three, $14.00 \mu\text{m}$ has four, 0.0140 s has three, 5300 kg has four in this book (but is considered ambiguous in general).

(b) 1.2×10^4 kg (or 12 Mg); 0.013 s (or 1.3×10^{-3} s or 1.3 ms).

Suppose you measure the mass and volume of some object to be $m = 1.2$ kg and $V = 0.123 \text{ m}^3$, and you wish to use these values to compute the object's mass density. If you substitute the measured values into Eq. 1.4 and carry out the division on your calculator, you get

$$\rho \equiv \frac{m}{V} = \frac{1.2 \text{ kg}}{0.123 \text{ m}^3} = 9.75609756 \text{ kg/m}^3. \quad (1.7)$$

You measured the mass to two significant digits and the volume to three significant digits, but your calculator suggests you have determined the density to a precision of better than one part in a billion! The rigorous way to deal with uncertainties in measurements is to recalculate any computed value by using the high and low uncertainties for each value in the calculation, which is a

time-consuming task. For this reason, we shall use two shortcuts. The first deals with multiplication and division:

When multiplying or dividing quantities, the number of significant digits in the result is the same as the number of significant digits in the input quantity that has the fewest significant digits.

In your calculation of the mass density in Eq. 1.7, the input quantity that has the fewest significant digits is the mass (two significant digits), and so the answer should be rounded to two significant digits:

$$\rho \equiv \frac{m}{V} = \frac{1.2 \text{ kg}}{0.123 \text{ m}^3} = 9.8 \text{ kg/m}^3. \quad (1.8)$$

Now suppose you determine the mass of two parts of an object and obtain 105 kg for one part and 0.01 kg for the other (three and one significant digit, respectively). Adding the two measured values yields a value containing five significant digits:

$$105 \text{ kg} + 0.01 \text{ kg} = 105.01 \text{ kg}. \quad (1.9)$$

Given that the precision in the value 105 kg is ± 0.5 kg, it makes no sense to report the final result to five significant digits. Rounding so that the number of significant digits in the sum is the same as the number of significant digits in the input value that has the fewest significant digits would yield 100 kg, which deviates from the actual value (around 105 kg) by significantly more than the precision in the individual measurements (± 0.5 kg and ± 0.005 kg). To solve this problem, whenever we add or subtract numerical values, we focus on the number of *decimal places*:

When adding or subtracting quantities, the number of decimal places in the result is the same as the number of decimal places in the input quantity that has the fewest decimal places.

For the addition in Eq. 1.9, the input quantity that has the fewest decimal places is 105 kg (zero decimal places), and so

$$105 \text{ kg} + 0.01 \text{ kg} = 105 \text{ kg}. \quad (1.10)$$

Bear in mind that *counting numbers* (that is, integers) are exact and so have an infinite number of significant digits. For example, the product of the width of two buttons each 15.7 mm wide is $2 \times 15.7 \text{ mm} = 31.4 \text{ mm}$ (not $3 \times 10^1 \text{ mm}$ because the 2 is a counting number).

Exercise 1.9 Significant digits and calculations

Calculate: (a) $f = a/(bc)$, where $a = 2.34 \text{ mm}^2$, $b = 54.26 \text{ m}$, and $c = 0.14 \mu\text{m}$; (b) $g = kt^3$, where $k = 1.208 \times 10^{-2} \text{ s}^{-3}$ and $t = 2.84 \text{ s}$; (c) $f + g$; (d) the sum of $b = 54.26 \text{ m}$ and $c = 1.4 \text{ mm}$; (e) $h = k(m - n)$, where $k = 1.252$, $m = 32.21$, and $n = 32.1$.

SOLUTION (a) I first need to convert the millimeters-squared in $a = 2.34 \text{ mm}^2$ to meters-squared and the micrometers in $c = 0.14 \mu\text{m}$ to meters:

$$2.34 \text{ mm}^2 \left(\frac{1 \text{ m}}{1 \times 10^3 \text{ mm}} \right)^2 = 2.34 \text{ mm}^2 \left(\frac{1 \text{ m}^2}{1 \times 10^6 \text{ mm}^2} \right) = 2.34 \times 10^{-6} \text{ m}^2$$

$$0.14 \mu\text{m} \left(\frac{1 \text{ m}}{1 \times 10^6 \mu\text{m}} \right) = 0.14 \times 10^{-6} \text{ m}.$$

(Continued)

Substituting the values given, I get

$$f = \frac{2.34 \times 10^{-6} \text{ m}^2}{(54.26 \text{ m})(0.14 \times 10^{-6} \text{ m})} = 0.30804,$$

which I must round to two significant digits, 0.31, because that is the number of significant digits in the value given for c .

(b) $g = (1.208 \times 10^{-2} \text{ s}^{-3})(2.84 \text{ s})^3 = 0.2767$, which I must round to three significant digits, 0.277, because of t .

(c) $f + g = 0.30804 + 0.2767 = 0.58474$, which I round to 0.58 because I must report only two decimal places, limited by $c = 0.14 \mu\text{m}$. Note that if I had added the rounded values for f and g , my result reported to two significant digits would not be 0.58: $f + g = 0.31 + 0.277 = 0.587$, which rounds to 0.59. Be careful—reporting the correct number of significant digits can be tricky.

(d) $b + c = 54.26 \text{ m} + 0.0014 \text{ m} = 54.26 \text{ m}$ (reported to two decimal places, limited by the 54.26 value).

(e) $h = 1.252(32.21 - 32.1) = 0.1$ (reported to one significant digit, limited by the 32.1 value).

The number zero requires special consideration. When some physical quantity is exactly zero, we denote this quantity by a zero *without units*. The speed of an object that is not moving is exactly zero and is therefore denoted by $v = 0$ (no units). However, if we measure a speed as zero to two significant digits, we write $v = 0.0 \text{ m/s}$ (note the units), implying that the actual value is zero to within 0.05 m/s.



1.14 (a) Express the circumference of a circle of radius $R = 27.3 \text{ mm}$ with the correct number of significant digits. (b) Let $a = 12.3$, $b = 3.241$, and $c = 55.74$. Compute $a + b + c$. (c) Let $m = 4.00$, $n = 3.00$, and $k = 7$ (exact). Compute $f = m^2/k$, $g = n^2/k$, and $f + g$.

1.8 Solving problems

You encounter problems every day. You are in a rush to get somewhere but can't find your car keys. You run out of flour while baking a birthday cake, and the supermarket is closed. Your flight is canceled on your way to a job interview. You want to buy those great new shoes, but there's no money in the bank. There is no formula for solving these or any other types of problems (if there were a formula, you wouldn't have a problem!). Because real-world problems defy one-shot solutions, we must break them down into smaller parts and solve them in steps. In this section we develop a four-step problem-solving strategy that will help you tackle a broad variety of problems (the procedure is summarized in the box "Solving problems").

Most physics problems are formulated in words. This is true not only of the problems in this book but also of questions you might have about the world around you. Standing on the rim of the Grand Canyon, you might wonder, If I drop a stone, how long does it take to hit the bottom?, or while watching the Olympics, you might wonder, Is there a physical limit to how high an athlete can jump?

1 GETTING STARTED

Because it's not clear which road leads you most efficiently to the answer to a given problem, the first step of our problem-solving strategy, *getting started*, is the most difficult one. It is therefore useful to begin with something you *can* do: Organize the information given and be sure you are clear on exactly what is asked for in the problem. To clarify the goal of the problem in your mind, it is crucial to begin by visualizing the situation: Sketch, describe in your own

words, list, and/or tabulate the main features of the problem to relieve yourself from having to hold all this information in memory. In making a sketch, be sure to follow the guidelines given in Section 1.5: Make your rendering as simple as possible and show all relevant numerical information in the sketch. Recasting your problem visually or verbally forces you to spell out what you want to accomplish and often automatically leads to ideas on how to solve the problem.

Once you have the information organized, ask yourself which concepts and principles apply. Throughout this book, we develop principle-specific procedures for solving problems. By determining which principles apply to a problem you are working on, you can identify which problem-solving procedures apply to your problem.

Finally, you must determine whether or not you have all the information necessary to solve the problem. Sometimes you may have to supplement the information provided with things you know about the situation at hand. Furthermore, because no problem can be formulated with absolute precision, you will often need to make a number of simplifying assumptions. This lack of precision is often a source of frustration for anyone beginning to solve physics problems. How do you know which assumptions are valid for a given problem? Do not let this question trouble you—as long as you are aware of the assumptions you make, you can always reexamine them once you have obtained an answer and then refine the assumptions and the solution.

2 DEVISE PLAN

Next you must *devise a plan* for solving your problem, which means spelling out what you must do to solve the problem. Are there any physical relationships or equations that you can apply to determine the information you are trying to obtain? A good plan is to outline the steps you need to take to obtain a solution. For some (but certainly not all) problems, these steps are carried out mathematically.

3 EXECUTE PLAN

In the third step, you *execute your plan* by following the steps you have outlined, substituting the information given and carrying out any mathematical operations necessary to isolate the quantity you wish to determine. In problems involving numerical values, you should solve for an algebraic answer first, waiting until the final step to substitute numerical values and obtain a numerical answer. The only exception to this algebraic-answer-first approach is that, in order to simplify your algebraic expressions, you should eliminate any quantities that are zero from them as soon as possible.

In the final step, substituting numerical values into your algebraic expression, do not forget to show the correct units on all numerical values and to carry these units through the calculation.

Once you have obtained an answer, you should check your work for these five important points (which you can remember via the acronym *VENUS*):

1. Vectors/scalars used correctly? As you will see in Chapter 2, the quantities we deal with in this book fall into two classes, vectors and scalars. Make sure you have expressed your answer in the appropriate form. (In this chapter all quantities are scalars.)
2. Every question asked in problem statement answered? Reread the problem statement to make sure you have completely answered every question.
3. No unknown quantities in answers? If your answer is an algebraic expression, be sure that every variable appearing in terms to the right of the equals sign has been given as a known quantity in the problem statement. Example: “A car moving at speed $v \dots$ ” means that v in this problem is a known quantity.

4. **Units correct?** Each numerical answer should be expressed as a number with the appropriate units for that quantity (Section 1.6). When your answer is an algebraic expression, be sure that the combined units of all terms to the right of the equals sign work out to be equivalent to the unit of the quantity to the left of the equals sign.
5. **Significant digits justified?** The number of significant digits reported in your answer should reflect the number of significant digits of the quantities given in the problem statement (Section 1.7).

As a reminder to yourself, put a checkmark beside each answer to indicate that you checked these five points.

4 EVALUATE RESULT

You may think you are done, but there is one final—and very important—step left: *Evaluate your answer*. In this step, you should reflect on your work, examine the result, and determine whether your answer is reasonable (or, if you can't tell whether or not it is reasonable, at least make sure that it is not unreasonable). The first thing you might try is seeing whether your answer conforms to what you expect based on your situation sketch, diagram, or other information given. If any part of your answer is unexpected, go back and check your math and any assumptions you made.

If your result is an algebraic expression or if your numerical answer comes directly from an algebraic expression, check whether or not that expression behaves as you expect with changes in physical quantities. For example, say you obtain the expression $v = k/t$ for the speed of an object and you know from experience that for this object the speed is zero for large values of t . If you let t go to infinity, your expression should indeed give the right answer: $v = k/\infty = 0$. If it didn't, you must go back and check your math and any assumptions you made.

Some problems can be solved in more than one way. If you solve your problem in two ways and obtain the same answer, you can be pretty sure your answer is correct. If you don't get the same answer from the two ways, you'd better go back and check your work!

Procedure: Solving problems

Although there is no set approach when solving problems, it helps to break things down into several steps whenever you are working a physics problem. Throughout this book, we use the four-step procedure summarized here to solve problems. For a more detailed description of each step, see Section 1.8.

1. **Getting started.** Begin by carefully analyzing the information given and determining in your own words what question or task is being asked of you. Organize the information by making a sketch of the situation or putting data in tabular form. Determine which physics concepts apply, and note any assumptions you are making.
2. **Devise plan.** Decide what you must do to solve the problem. First determine which physical relationships or equations you need, and then determine the order in which you will use them. Make sure you have a sufficient number of equations to solve for all unknowns.
3. **Execute plan.** Execute your plan, and then check your work for the following five important points:

- Vectors/scalars used correctly?
- Every question asked in problem statement answered?

- No unknown quantities in answers?
- Units correct?
- Significant digits justified?

As a reminder to yourself, put a checkmark beside each answer to indicate that you checked these five points.

4. **Evaluate result.** There are several ways to check whether an answer is reasonable. One way is to make sure your answer conforms to what you expect based on your sketch and the information given. If your answer is an algebraic expression, check to be sure the expression gives the correct trend or answer for special (limiting) cases for which you already know the answer. Sometimes there may be an alternative approach to solving the problem; if so, use it to see whether or not you get the same answer. If any of these tests yields an unexpected result, go back and check your math and any assumptions you made. If none of these checks can be applied to your problem, check the algebraic signs and order of magnitude.

If none of the evaluation methods can be applied to your answer, you should at least verify that there is nothing obviously wrong with it by checking that the algebraic signs you obtain make sense and that any numerical value has the correct order of magnitude. For example, if the algebraic sign of your solution tells you that an object moves to the left when you know it should move to the right, you know your answer can't be right. You can find some techniques for estimating orders of magnitude in the next section. Using these techniques, you should be able to obtain reasonable upper and lower bounds for your answer. Provided your answer does not lie outside this range, you know there is not something obviously wrong with it.

With the exception of one-step exercises that are applications of procedures, the examples in this book follow this four-step procedure.

Example 1.10 Length of string

A certain cylinder is 15 in. long and has a circumference of 4.0 in. When one end of a piece of string is attached to the left end of the cylinder and the string is then wound exactly five times around, the string just reaches to the cylinder's right end. What is the length of the string?

1 GETTING STARTED I begin by making a sketch of the situation (Figure 1.19a). The shortest length is going to be obtained when the string is wound uniformly, with each of the five windings running one-fifth of the length of the cylinder. I need to find a way to determine the length of this spiraling string. This appears to be a geometry problem, but the surface of the cylinder is curved, not flat.

2 DEVISE PLAN If I imagine drawing a straight line between the endpoints of the string, cutting the cylinder surface along that line, and then flattening the cylinder out, I obtain the situation shown in Figure 1.19b. Now the solution is clear: The total length of the string is the sum of the hypotenuses of five identical triangles.

3 EXECUTE PLAN The width of each triangle is equal to one-fifth of the length of the cylinder: $(15 \text{ in.})/5 = 3.0 \text{ in.}$ The height of each triangle is equal to the circumference of the cylinder (4.0 in.). With these values, I can now use the Pythagorean theorem to determine the length ℓ of the hypotenuse of each triangle:

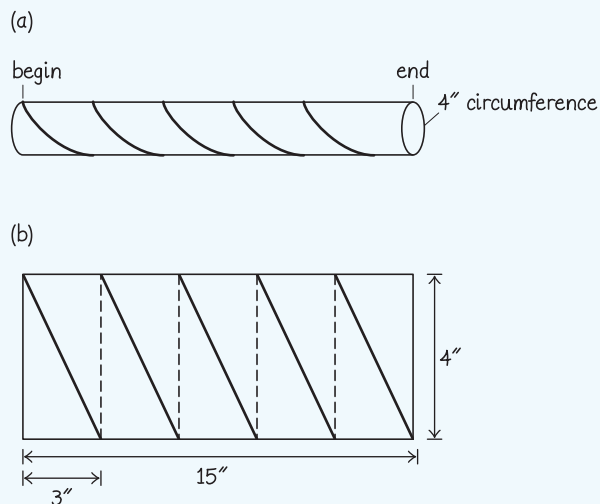
$$\ell = \sqrt{(3.0 \text{ in.})^2 + (4.0 \text{ in.})^2} = 5.0 \text{ in.}$$

The length of the string is equal to five times this value, or 25 in. ✓*

4 EVALUATE RESULT The answer I obtain should be larger than the string length I would obtain if the string were wound around the cylinder in one place, which would give a string length of five times the circumference: $5 \times 4.0 \text{ in.} = 20 \text{ in.}$ My answer should be smaller than what I would get if the string were wound five times around the cylinder at one end and then run along the cylinder length to the other end, which would give a string length of five times the circumference plus the length of the cylinder: $(5 \times 4.0 \text{ in.}) + 15 \text{ in.} = 35 \text{ in.}$ The answer I obtained does indeed lie between these values, giving me confidence that it is correct.

*The checkmark shows that I checked my answer regarding the five important details listed on page 21: Vector/scalar: all quantities scalar; Every question answered: yes; No unknown quantities in answers: no; Units correct: yes, inches; Significant digits: everything given to two significant digits.

Figure 1.19



1.9 Developing a feel

Making order-of-magnitude estimates is possibly the most important skill to retain from a physics course because this is a skill you will use long after you leave your last classroom, no matter what you plan for a career. Economists, for instance, carry out order-of-magnitude estimates to predict market trends, and business owners might carry out such estimates to validate a business plan, to name just two possibilities.

Here in this physics course, to get back to the present, order-of-magnitude estimates allow you to develop a feel for a problem before you try to solve it. They also allow you to validate your answer: If your order-of-magnitude estimate and your answer disagree, you know to go back and reevaluate your work.

Order-of-magnitude estimates involve determining relationships between known and unknown quantities, making gross oversimplifications, combining assumptions and ideas, and even outright guessing. In general, there are multiple ways to obtain such estimates. The uncertainty in how to carry out an estimate is intimidating at first, and the lack of accuracy may appear unscientific to you. It isn't—estimation is an important aspect of science, and it is a great way to obtain physical insight and at the same time train your analytical and outside-the-box thinking skills.

For example, suppose we need to determine the number of cells in the human body. That we can't see individual cells tells us they must be very small, and therefore the number is probably very large. We could *guess* the number of cells, but the answer would probably be meaningless because we have no way of assessing its accuracy without looking it up somewhere. A billion? Ten billion? Hundred billion? What do you think?

To obtain an order-of-magnitude estimate, the first thing to do is to think of a strategy. For the number of cells in the human body, for example, if we knew the volume of a typical human body and the volume of a typical cell, we could divide the former by the latter to obtain our answer. Knowing neither volume, however, we need to estimate both.

There are several ways to estimate the volume of the human body. One way is to model the body as a cylinder (Figure 1.20). For the height of the cylinder, we take the height of a typical human, say $h_{\text{human}} \approx 1.7$ m. The symbol \approx indicates that this value is an approximation. For the diameter, we take $d_{\text{human}} \approx 0.2$ m. That makes the diameter less than the left-to-right thickness of the body but greater than the front-to-back thickness, which should be in the right ballpark. This gives a volume of

$$V_{\text{human}} = \left(\frac{1}{4} \pi d_{\text{human}}^2 \right) h_{\text{human}} \approx 0.05 \text{ m}^3.$$

Another way is to realize that the human body has roughly the same mass density as water—about 1 kg/L, which is

$$1 \text{ kg/L} \left(\frac{1 \text{ L}}{1 \times 10^{-3} \text{ m}^3} \right) = 1000 \text{ kg/m}^3.$$

The average human has a mass of about 70 kg, which corresponds to 70 L of water, and thus $V_{\text{human}} \approx (70 \text{ L})(1 \times 10^{-3} \text{ m}^3/1 \text{ L}) = 0.07 \text{ m}^3$, which is in the same ballpark as the estimate we got using the body-as-cylinder approach.

You may feel that there is too big a difference between 0.05 m^3 and 0.07 m^3 . Indeed, if you were shopping for clothing, you'd want your clothing to fit your body better than that. However, here we are interested in only the order of magnitude. Either estimate is good enough for this purpose because for both the order of magnitude is the same: $0.05 \text{ m}^3 = 5 \times 10^{-2} \text{ m}^3$, which rounds to $10 \times 10^{-2} \text{ m}^3$, and so the order of magnitude is 10^{-1} m^3 ; $0.07 \text{ m}^3 = 7 \times 10^{-2} \text{ m}^3$, which also rounds to $10 \times 10^{-2} \text{ m}^3$, giving the same order of magnitude: 10^{-1} m^3 .

Next we need to estimate the volume of a typical human cell. You may remember from biology class that the diameter of a typical cell is around $20 \mu\text{m} = 20 \times 10^{-6} \text{ m}$. Or, if you never learned cell sizes, you may recall that you can clearly see cells under a microscope and that a microscope can see objects as small as 10^{-6} m . To see the structure of a cell clearly under a microscope, the cell would need to be, say, at least 50 times the minimum size the microscope

Figure 1.20 To determine the volume of the human body, we approximate the body as a cylinder.



can handle, making our cell-diameter estimate 50×10^{-6} m. Although you may think there is a big difference between 20×10^{-6} m and 50×10^{-6} m, these two numbers are only a factor of 2.5 apart, which is much less than a factor of 10 (“one order of magnitude”).

Cells viewed under a microscope tend to spread out on the glass and are thus less high than they are wide. Let’s for simplicity assume they are disk-shaped and that their height is about one-quarter of their diameter. That means they have a diameter of 20×10^{-6} m and a height of 5×10^{-6} m, and so $V_{\text{cell}} = (\frac{1}{4}\pi d_{\text{cell}}^2)h_{\text{cell}} \approx 1.6 \times 10^{-15}$ m³. The number of cells is thus $V_{\text{human}}/V_{\text{cell}} \approx (0.05 \text{ m}^3)/(1.6 \times 10^{-15} \text{ m}^3) \approx 3.1 \times 10^{13}$. Given the roughness of the estimates and assumptions, I give the final answer as an order of magnitude: 10^{14} (about 100 trillion cells).

I can be quite confident that the number of cells is somewhere between 10^{13} (10 trillion) and 10^{15} (1000 trillion), both substantially greater than the largest guess I offered earlier (100 billion = 10^{11}). The difference between that largest guess and the order-of-magnitude estimate is that I have no idea how much confidence to place in the guess, whereas I can feel reasonably certain about the result of my order-of-magnitude estimate.

You may think that this exercise is outrageous and that it is impossible to have faith in any order-of-magnitude value because it requires so many assumptions and estimates. However, the fact that we combine so many assumptions and estimates is precisely why the method works: We are unlikely to be either too high on every estimate or too low on every estimate. Instead, we unwittingly overestimate some values and underestimate others, and the over- and underestimates tend to cancel out. The result is that the final answer very likely is in the right ballpark. What is more important is that, in the process of obtaining an order-of-magnitude value, we develop insights and some feeling for the quantities involved.

Figure 1.21 shows in graphical form the process by which we obtained our estimate of the number of cells N_{cell} in the human body. We got to our answer through a combination of *devising a strategy* (using a ratio of volumes to determine the number of cells), *simplifying* (treating the human body as a cylinder and the cell as a disk), *estimating* (the diameter of the human body), and using snippets of *knowledge*. Figure 1.21 also shows why this approach can be intimidating at first: To determine a single target unknown, N_{cell} , we have to divide the problem into a larger number of questions, an approach that makes the number of unknowns increase initially! The point of this dividing, of course, is to arrive at values we know (or can guess).

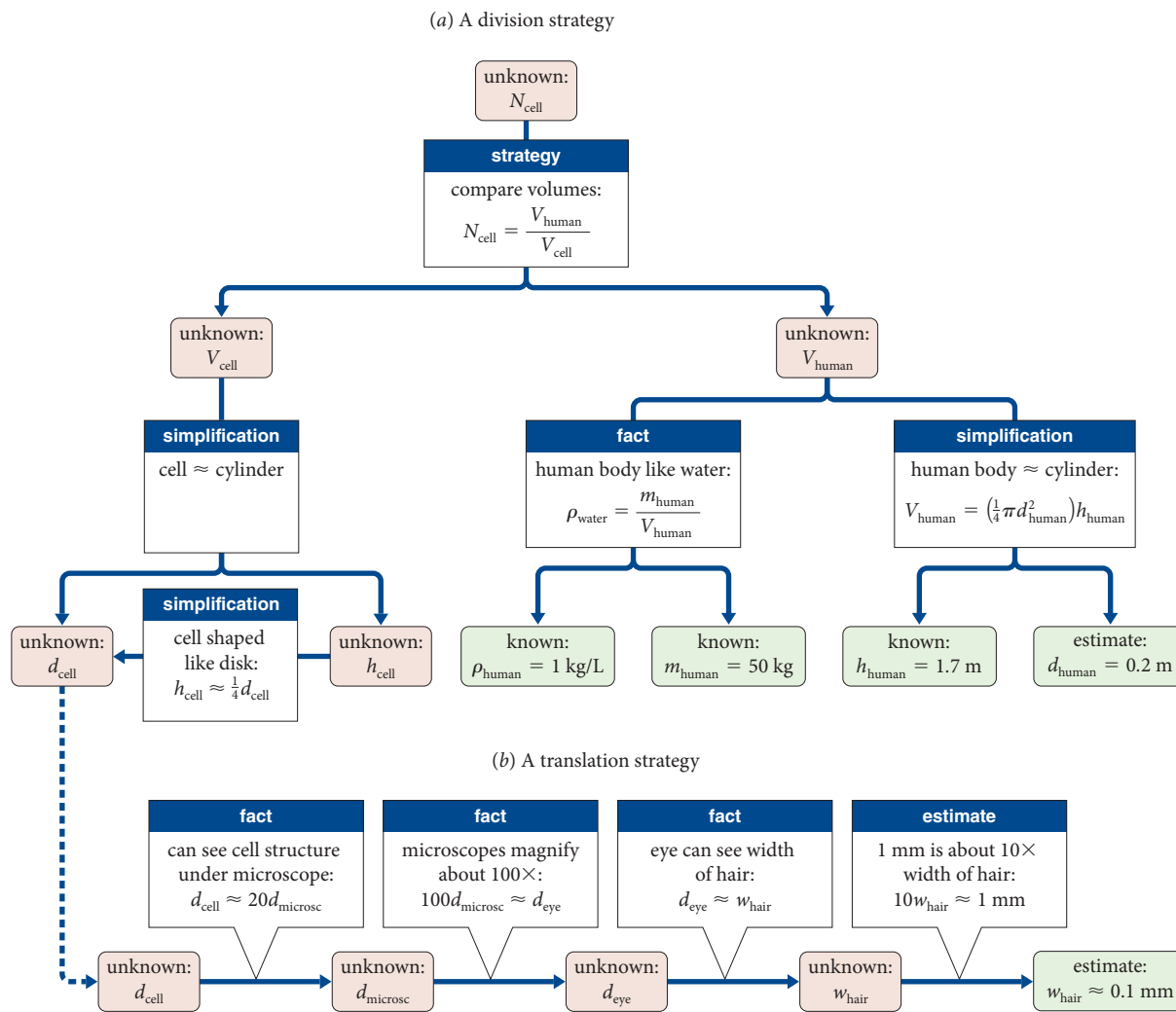
The diagram in Figure 1.21a reduces the problem of obtaining N_{cell} to one of obtaining another value, d_{cell} . If you know the value of d_{cell} from your biology course, you are done. If you don’t know the value, you can *translate* the problem into another one, as illustrated in Figure 1.21b. If you get stuck, guess upper and lower bounds for your unknowns, use the middle value to obtain an answer, and then reevaluate to see how your *guesstimate* affects the final answer.

Making order-of-magnitude estimates requires practice. As you go through the exercise more often, you’ll gain confidence, and the process will come more naturally. An important point is not to worry too much about accuracy when working out orders of magnitude: A factor of ten doesn’t matter. You’ll be surprised how far you can get with this technique, some resourcefulness, and commonly available knowledge.



1.15 To work an order-of-magnitude problem, you need the value of the mass density of water, but you don’t know what that value is. Design either a translation strategy (Figure 1.21b) or a division strategy (Figure 1.21a) for making an order-of-magnitude estimate of this mass density.

Figure 1.21 The process by which we estimated the number of cells in the human body.



Exercise 1.11 Selling books

Consider writing a book for high-school graduates in the United States who are planning to take a year off—sometimes called a gap year—before enrolling in college. There are currently no similar books. Estimate how many such books might be purchased annually.

SOLUTION I can estimate the number of books sold by estimating the percentage of college-bound high-school graduates who take a gap year. I know two things: (1) Not all graduates do this, and (2) there is a rising interest in doing it. I estimate that 5% of high-school graduates take a gap year before going to college, but I still have to estimate the number of college-bound high-school students in the United States. I know that approximately 50% of high-school graduates go on to college, but how many high-school graduates are there each year?

If I assume every person in the country graduates from high school when she or he is 18, I've reduced this part of the problem to determining the number of 18-year-olds in the country. I know that the total population is 300 million and that the average life span is about 75 years. If I assume that age is uniformly distributed, then 1/75 of the U.S. population should be 18 years old at any given instant (this is an oversimplification but is good enough for an order-of-magnitude estimate). So the number of high-school graduates in a given year is $(1/75)(3 \times 10^8) = 4 \times 10^6$, and 5% of this is 2×10^5 . Thus the number of students taking a gap year each year is approximately 200,000, which strikes me as a reasonable number. If 5% of these students buy the proposed book, about 10,000 copies will be purchased each year.

QUANTITATIVE TOOLS

1.16 Make an order-of-magnitude estimate of the mass of Earth in kilograms.

Chapter Glossary

SI units of physical quantities are given in parentheses.

Absolute time The notion that time is the same for all observers in the universe, regardless of their location or motion.

Atom A basic building block of matter.

Avogadro's number The number of atoms or molecules in 1 mol:

$$N_A = 6.022 \times 10^{23}. \quad (1.2)$$

See also *mole*.

Change The transition from one state to another.

Density A measure of how much of some substance there is in a given volume. The *number density* (m^{-3}) is the number of objects per volume:

$$n \equiv \frac{N}{V}. \quad (1.3)$$

The *mass density* (kg/m^3) is the amount of mass per volume:

$$\rho \equiv \frac{m}{V}. \quad (1.4)$$

Electron A subatomic particle manifesting itself in a cloud around atomic nuclei.

Hypothesis A tentative explanation of observations to be used as a starting point for further investigation.

Law A description of a relationship between observable quantities that manifests itself in recurring patterns of events.

Length ℓ (m) A distance or extent in space.

Meter (m) The SI base unit of length, defined as the distance traveled by light in vacuum in $1/299,792,458$ of a second.

Model A simplified conceptual representation of a phenomenon.

Mole (mol) The SI base unit for measuring the quantity of a given substance, defined as the number of atoms in 12×10^{-3} kg of carbon-12, which is measured to be 6.022×10^{23} . See also *Avogadro's number*.

Order of magnitude A value rounded off to the nearest power of ten.

Physical quantity A physical property that can be measured and that is expressed as the product of a numerical value and a physical unit.

Principle of causality Whenever an event A causes an event B, all observers see event A happening first.

Proton, neutron Subatomic particles residing in atomic nuclei.

Scientific method An iterative process for going from observations to theories validated by experiments.

Second (s) The SI base unit of time, defined as the duration of 9,192,631,770 periods of certain radiation emitted by cesium atoms.

SI units Units of measure in the International System, used in science and engineering.

Significant digits Digits in a numerical value that are reliably known.

Symmetry An object exhibits symmetry when certain operations can be performed on it without changing its appearance.

Theory A well-tested explanation of a natural phenomenon in terms of more basic processes and relationships.

Time t (s) A physical quantity that allows us to determine the sequence of related events.

Universe The totality of matter and energy combined with the space and time in which all events happen.